Triads and Triadic Relations

John F. Sowa

Responding to “The Necessity of Genuine Triadic Relations” by William McCurdy

APA Pacific Division Meeting, March 2018
Some Triads Are Not Relations

Charles Sanders Peirce never claimed that triadic relations could not be defined in terms of dyadic relations.

His claim was about graphs, not about relations:

- For existential graphs, he coined the word 'teridentity' for a 'ligature' of three lines of identity — the equivalent of \((x = y = z)\).
- A triadic connection in a graph could be either a triadic relation or a teridentity.
- If a triadic relation is redefined in terms of dyadic relations, the graph would still contain a node with a triadic connection.
- That node may be a teridentity linked to three dyadic relations.
- Or it may be a subgraph that contains a nominalized version of the original triadic relation that is linked to the same three nodes as that original relation.
Relating Different Ontologies

Two conceptual graphs for the sentence, “Sue gives a child a book.”

Any triadic relation may be replaced by three dyadic relations. But the topology of the graph still contains a triadic connection. That fact is harder to see in a linear notation notation for logic:

\[(\exists x)(\exists y)(\text{Person}(Sue) \land \text{Child}(x) \land \text{Book}(y) \land \text{Gives}(Sue,x,y)).\]

\[(\exists x)(\exists y)(\exists z)(\text{Person}(Sue) \land \text{Child}(x) \land \text{Book}(y) \land \text{Give}(z) \land \text{Agnt}(z,Sue) \land \text{Rcpt}(z,x) \land \text{Thme}(z,y)).\]
Graphs for Representing Logic

Every language, natural or artificial, has four kinds of signs:
1. Symbols for entities, concepts, relations, functions...
2. Indices (labels, names, pronouns) for internal and external references.
3. Icons that connect signs in graphs (strings, trees, or networks).
4. Icons that group related graphs in contexts.

Peirce designed existential graphs for representing “the atoms and molecules of logic”.

Conceptual graphs are a version of logic based on existential graphs and the semantic networks used in artificial intelligence.

Conceptual Graph Interchange Format (CGIF) is one of three standard dialects in the ISO/IEC 24707 standard for Common Logic.

The Existential Graph Interchange Format (EGIF) is a linear notation for EGs that can be mapped to and from CGIF.

EGIF is useful as a compact notation that can convince skeptics that EGs can be just as formal as any linear notation for logic.
Mapping CGIF to EGIF

Two conceptual graphs for the sentence, “Sue gives a child a book.”

CGIF for the CG on the left:
[Person: Sue *x] [Child: *y] [Book: *z] (Gives x y z)
EGIF: (Person *x) (HasName x 'Sue') (Child *y) (Book z) (Gives x y z)

CGIF for the CG on the right:
[Person: Sue *x] [Child: *y] [Book: *z] [Give *w] (Agnt w x) (Rcpt w y) (Thme w z)
EGIF: (Person *x) (HasName x 'Sue') (Child *y) (Book *z) (Give *w) (Agnt w x) (Rcpt w y) (Thme w z)
Obligatory and Optional Relations

For many words, some relations may be obligatory or optional:

- For the verb 'give', linguists distinguish three obligatory semantic relations: agent (Agnt), recipient (Rcpt), and Theme (Thme).
- These semantic relations are mapped to and from the syntactic relations of Subject, Indirect Object, and Direct Object.
- In the sentence “I gave at the office”, there is an optional relation for location (Loc), but the obligatory relations for recipient and theme are missing.

When an obligatory relation is missing, some axiom is violated.

- In EG or EGIF, the axiom may be stated as an If-Then rule.
- [If (Give *w) [Then (Agnt w *x) (Rcpt w *y) (Thme w *z) ] ]
- In English: “If there is an act of giving w, then there is an agent x of w, a recipient y of w, and a theme z of w.”
- For the example “I gave at the office”, this axiom would imply that the agent is the person mentioned by the pronoun 'I', and that there should be an implicit recipient and theme in the context.
Mapping EGIF to EG

Two existentional graphs for the sentence “Sue gives a child a book.”

EGIF for the EG on the left:
(Person *x) (HasName x 'Sue') (Child *y) (Book z) (Gives x y z)

EGIF for the EG on the right:
(Person *x) (HasName x 'Sue') (Child *y) (Book *z) (Give *w)
(Agnt w x) (Rcpt w y) (Thme w z)

The node that represents x is a degenerate teridentity. The node for w, which has four links, may be replaced by two teridentities.
Operations on EGs

Replacing a tetridentity by two teridentities:

The tetridentity is replaced by two teridentities. The corresponding EGIF replaces \( w \) with \( u \) and \( v \) plus a coreference node \( [u *v] \):

\[
\text{(Person *x)} \ (\text{HasName x 'Sue'}) \ (\text{Child *y}) \ (\text{Book *z}) \ (\text{Give *u}) \\
(\text{Agnt u x}) \ [u *v] \ (\text{Rcpt v y}) \ (\text{Thme v z})
\]

The node that represents \( x \) is a degenerate teridentity. The following two statements are independent of one another:

- “Something is a person” : (Person *x)
- “Something has the name 'Sue’” : (HasName *x 'Sue')

Either one may be deleted without affecting the other.
Peirce introduced event semantics about 80 years before Davidson, but nominalists such as Strawson (1992) continued to object.

As an example, Strawson complained that the logic for “John kissed Mary in the garden at midnight” would be far too complex:

\[(\exists e)(\exists g)(\exists t) (\text{kiss}(e) \land \text{garden}(g) \land \text{midnight}(t) \land \text{agent}(e,\text{John}) \land \text{theme}(e,\text{Mary}) \land \text{location}(e,g) \land \text{pointInTime}(e,t))\].

But the conceptual graph shown above is quite simple, and the EG would be similar.

For both, the agent and theme relations are obligatory, and the location and point in time relations are optional.
Peirce’s Semiotic

<table>
<thead>
<tr>
<th>1. Quality</th>
<th>2. Indexicality</th>
<th>3. Mediation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Material</strong></td>
<td><strong>Relational</strong></td>
<td><strong>Formal</strong></td>
</tr>
<tr>
<td>Mark</td>
<td>Token</td>
<td>Type</td>
</tr>
<tr>
<td>A quality which is a sign.</td>
<td>An actual existent thing or event which is a sign.</td>
<td>A principle, habit, or law which is a sign.</td>
</tr>
<tr>
<td>Icon</td>
<td>Index</td>
<td>Symbol</td>
</tr>
<tr>
<td>Refers by virtue of some similarity to object.</td>
<td>Refers by virtue of being affected by object.</td>
<td>Refers by virtue of some law or association.</td>
</tr>
<tr>
<td>Predicate</td>
<td>Proposition</td>
<td>Argument</td>
</tr>
<tr>
<td>A sign of qualitative possibility.</td>
<td>A sign of actual existence.</td>
<td>A sign of law.</td>
</tr>
</tbody>
</table>

Quality may be expressed by a modad, indexicality by a dyad, but mediation requires an essential triad (a relation or its nominalization). Any version of logic must represent them in one way or another.
Reduction to a Dyadic Predicate

Quine (1953) showed how to eliminate n-adic relations for n>2: *

- For any theory $\Theta$ about a universe of discourse $U$, translate $\Theta$ to a theory $\Theta'$ about an enlarged universe $U'$:
  - Every predicate in $\Theta$ with three or more arguments in $U$ is replaced by a special predicate $F$ in $\Theta'$ with just two arguments in $U'$.
  - There is a truth-preserving mapping between statements of $\Theta$ and $\Theta'$.

But truth values are just one aspect of meaning:

- Any triadic connection among entities of $U$ would be preserved in $U'$.
- But the convoluted mapping from $U$ to $U'$ may obscure what people had intended to say about the theory $\Theta$ and its applications.
- The people who defined $\Theta$ may have also specified a mapping of $U$ to and from some aspect of the world that was important for them.
- But the mapping from $U$ to $U'$ would make it difficult for them to interpret statements about $U'$ without mapping them back to $U$.

Therefore, Quine’s reduction is interesting, but irrelevant.

Remarks by Two Logicians and a Poet

Charles Sanders Peirce:
It is easy to speak with precision upon a general theme. Only, one must commonly surrender all ambition to be certain. It is equally easy to be certain. One has only to be sufficiently vague. It is not so difficult to be pretty precise and fairly certain at once about a very narrow subject.

Alfred North Whitehead:
Human knowledge is a process of approximation. In the focus of experience, there is comparative clarity. But the discrimination of this clarity leads into the penumbral background. There are always questions left over. The problem is to discriminate exactly what we know vaguely.

Robert Frost:
I've often said that every poem solves something for me in life. I go so far as to say that every poem is a momentary stay against the confusion of the world.... We rise out of disorder into order. And the poems I make are little bits of order.

Peirce:  Do not block the way of inquiry.

Whitehead:  We must be systematic, but we should keep our systems open.
Some philosophers who tried to block the way of inquiry: Frege, Russell, Carnap, Quine, Strawson... *

Carnap’s strongest denunciation: “That’s poetry!”

Clarence Irving Lewis (1960), as quoted by Hao Wang:
It is so easy... to get impressive 'results' by replacing the vaguer concepts which convey real meaning by virtue of common usage by pseudo precise concepts which are manipulable by 'exact' methods — the trouble being that nobody any longer knows whether anything actual or of practical import is being discussed.

Whitehead, introducing Russell for the William James Lectures:
I am pleased to introduce my good friend Bertrand Russell.
Bertie thinks that I am muddle-headed, but then, I think that he is simple-minded.

Peirce, criticizing Ernst Mach:
Find a scientific man who proposes to get along without any metaphysics... and you have found one whose doctrines are thoroughly vitiated by the crude and uncriticized metaphysics with which they are packed.

* For references, see signproc.pdf, csp21st.pdf, worlds.pdf (URLs in next slide).
Related Readings


From existential graphs to conceptual graphs: http://jfsowa.com/pubs/eg2cg.pdf


