

Contexts in Language and Logic

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Outline

1. Context in language understanding

Literally, a context is text that accompanies a text. More generally, the context may be any background knowledge that helps explain a text.

2. Situation semantics

Situation semantics (Barwise and Perry 1983) is a version of context theory that was developed at Stanford (CSLI).

3. Representing contexts in logic

Algebraic logics can represent contexts, but visual information is an important supplement or extension. The work on logic and related issues by C. S. Peirce. The IKL extensions to Common Logic.

These slides were first presented on 25 October 2017 in a telecon sponsored by Ontolog Forum. An [audio recording](#) is available, but it starts abruptly at slide #3. Later, more slides were added for other talks, but none were deleted.

1. Context in Language Understanding

Observation by the linguist Zellig Harris:

- “We understand what other people say through empathy – imagining ourselves to be in the situation they were in, including imagining wanting to say what they wanted to say.”

Four kinds of context: text or discourse; physical situation; shared background; and goals or intentions of the participants.

Ambiguity in word senses: *My dog bit the visitor’s ear.*

- From knowledge about the size of dogs: a dachshund is unlikely.
- But if the visitor was in the habit of bending over to pet a dog, it might even be a chihuahua.

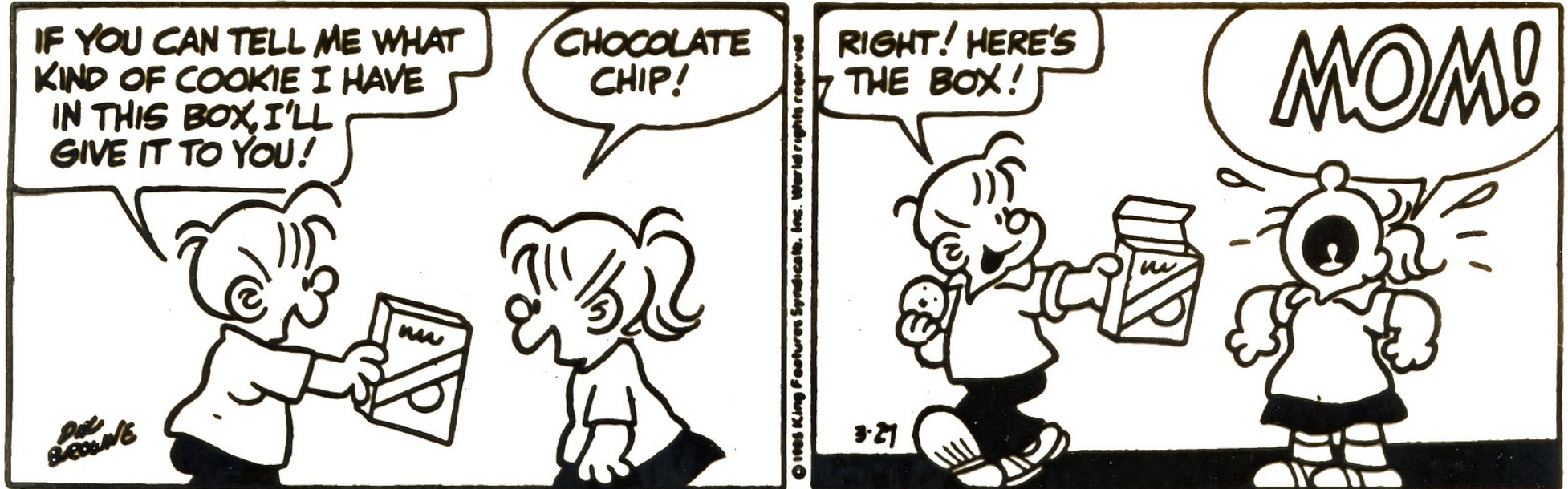
Ambiguity in syntax: *The chicken is ready to eat.*

- From knowledge about typical food: the chicken was cooked.
- If the word *chicken* were replaced with *dog*, one might assume that the dog was begging for food.
- But people in different cultures may make different assumptions.

There is no limit to the amount or kind of relevant context.

Context in Language

Hi & Lois

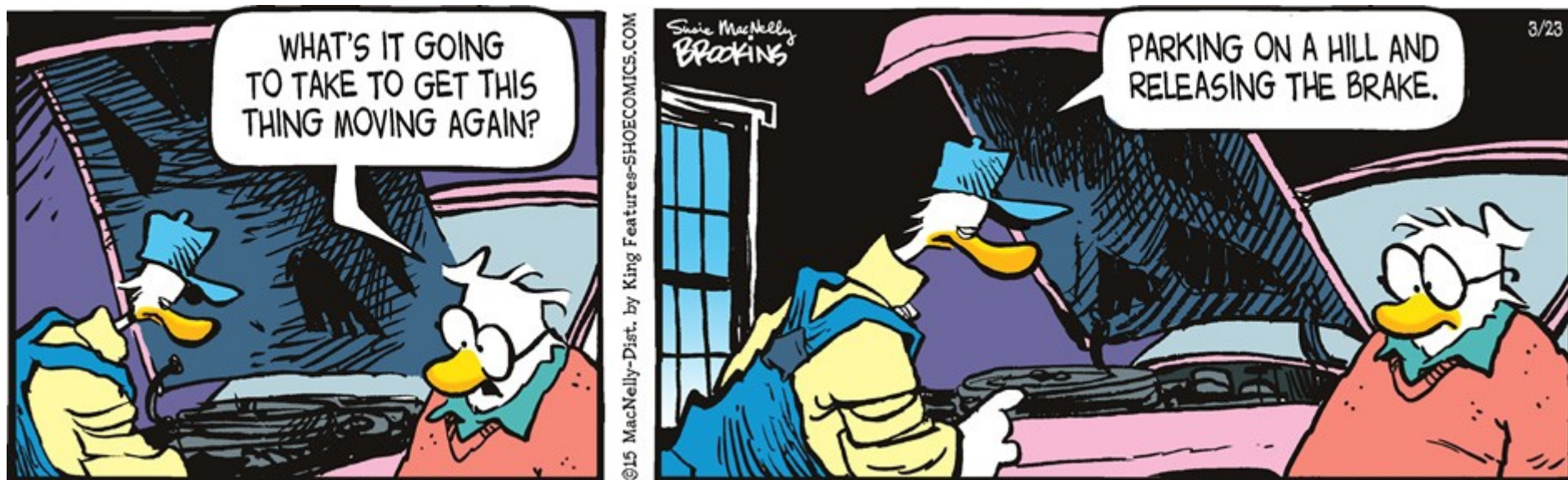


Four kinds of context: The text or discourse; the situation; common background knowledge; and the intentions of the participants.

Linguistics: Parse the sentences, resolve the referents of noun phrases, and determine the literal meaning of the text.

Pragmatics: Determine the implications by relating the meaning to the situation, the background knowledge, and the intentions.

Using the Context in NLP



Syntax is easy: Parse the question and the answer.

Semantics is harder: Use the context to

- Recognize the situation type and the roles of the two participants,
- Relate the word 'thing' to the car that is in a garage,
- Relate the verbs 'take' and 'move' to the situation,
- Apply the laws of physics to understand the answer.

Pragmatics is the hardest: Determine the intentions of the participants and their implications for the irony and humor.

* Source of cartoon: search for 'moving' at <http://www.shoecomics.com/>

Child Reasoning in Context

A mother talking with her son, about 3 years old: *

Mother: *Which of your animal friends will come to school today?*

Son: *Big Bunny, because Bear and Platypus are eating.*

The mother looks in his room, where the stuffed bear and the platypus are sitting in a chair and “eating”.

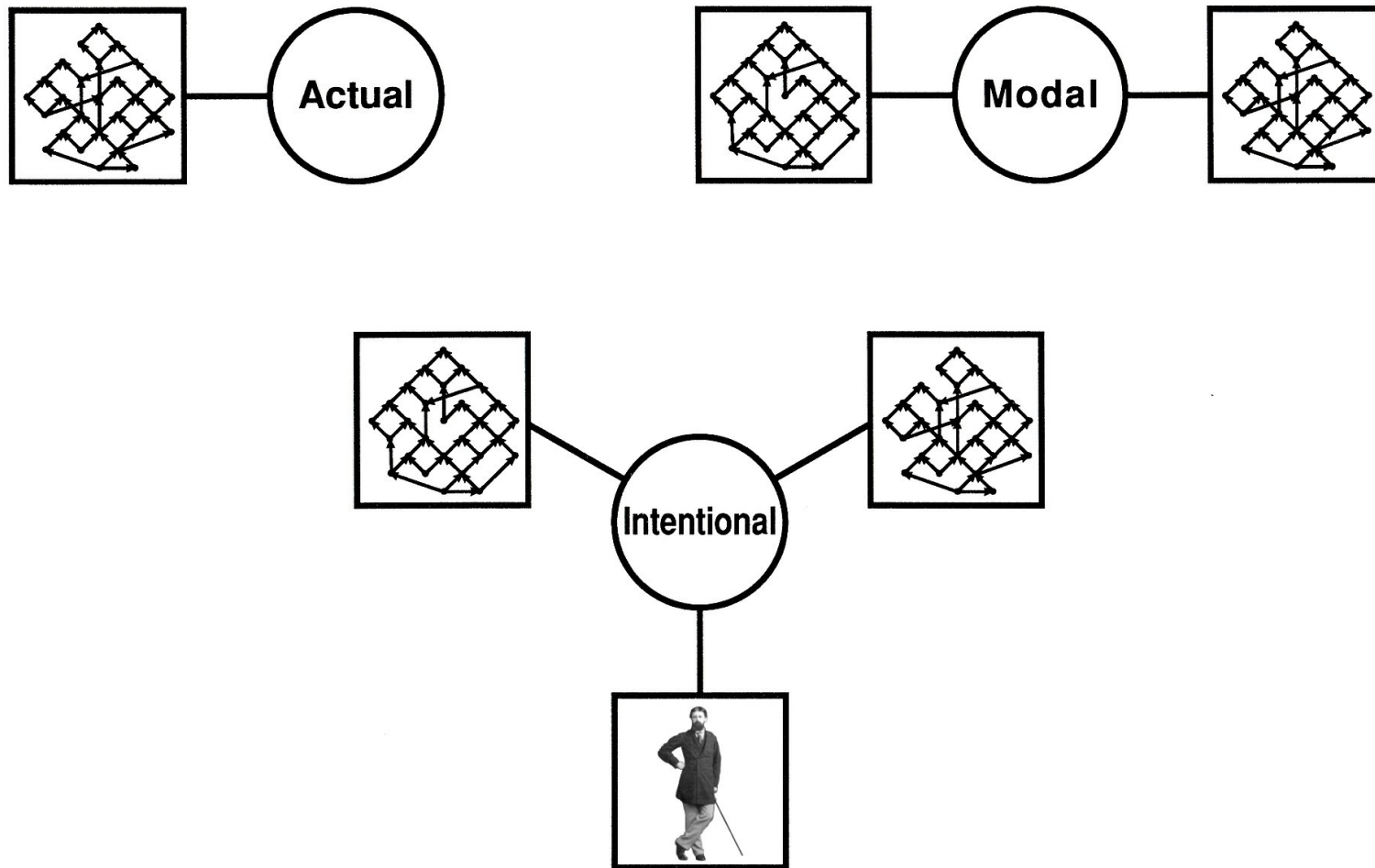
The child had related the sentences to the situation:

- **The bear and the platypus are eating.**
- **Eating and going to school cannot be done at the same time.**
- **Big Bunny isn't doing anything else.**
- **Therefore, Big Bunny is available.**

The child could reason about the context and even about the intentions that he attributed to his stuffed animals.

* Reported by the father, the psychologist Gary Marcus, in an interview with Will Knight (2015)
<http://www.technologyreview.com/featuredstory/544606/can-this-man-make-ai-more-human/#comments>

Actual, Modal, and Intentional Contexts



Three kinds of contexts, according to the source of knowledge:

- **Actual:** Something factual about the world.
- **Modal:** Something possible, as determined by some hypothesis.
- **Intentional:** Something an agent believes, desires, or intends.

Nested Situations

The three situations may be described as actual, modal, or intentional.

1. Actual: *Pierre is thinking of Marie, who is thinking of him.*

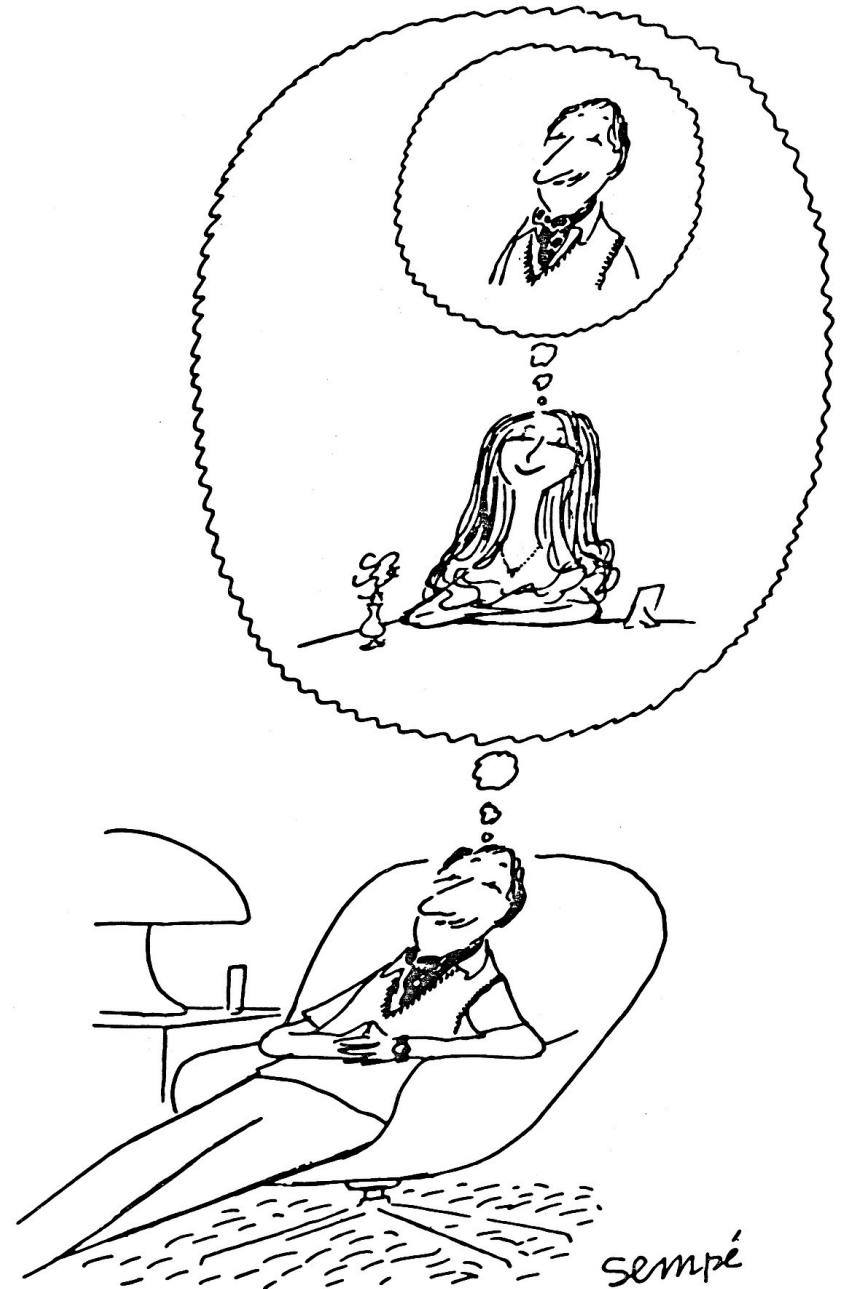
2. Modal: *Pierre is thinking of Marie, who might be thinking of him.*

3. Intentional: *Pierre thinks that Marie is thinking of him.*

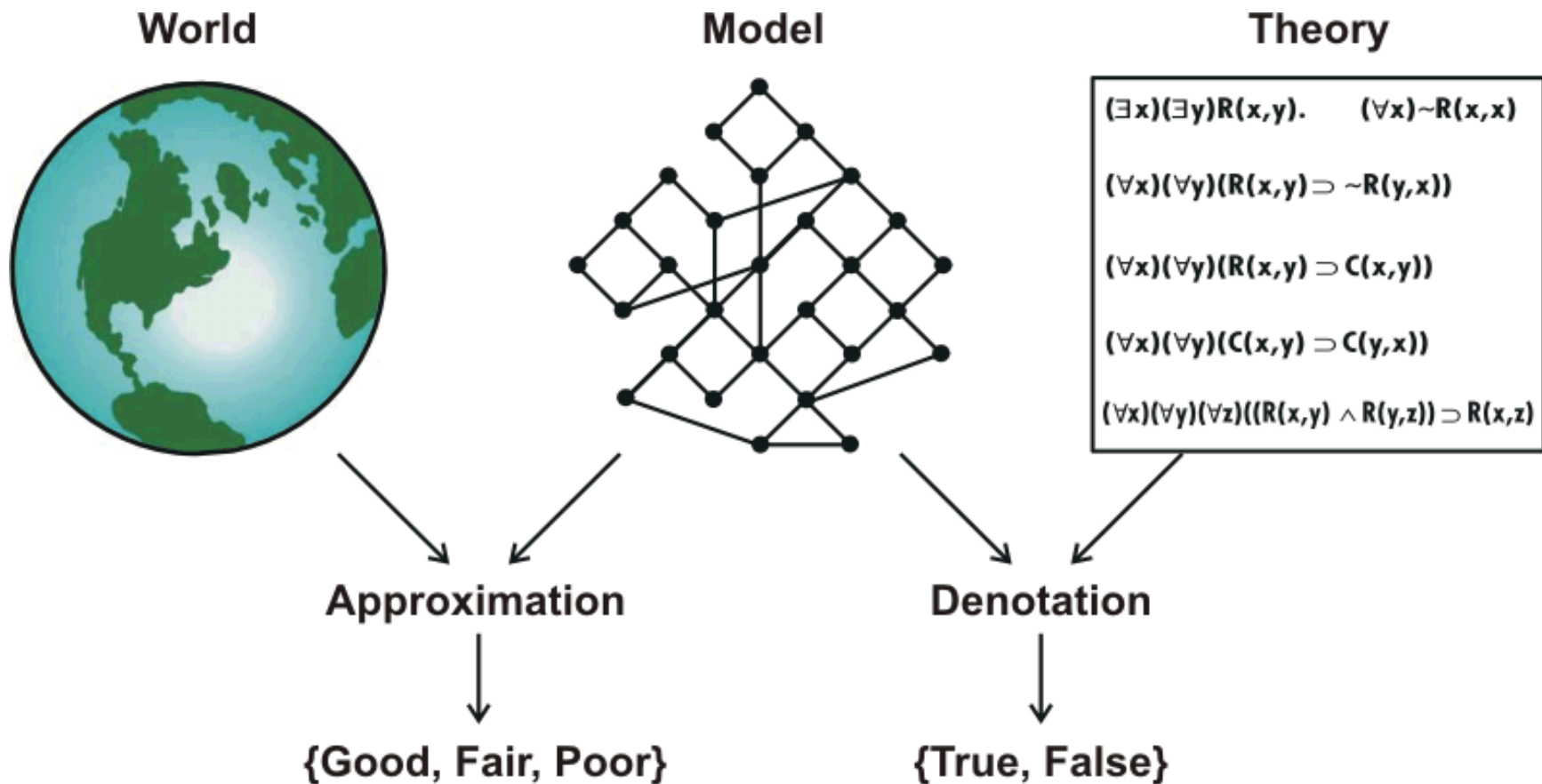
In #1, Pierre assumes that both clauses are true, but he may be mistaken.

In #2, the first clause is true, but the second may be true or false.

In #3, Pierre assumes or wishes that his thought is true, but it may be false.



Models of Worlds, Real or Possible



A Tarski-style model evaluates axioms of a theory in terms of a world, which may be described by a set, a network, or a database of facts. For modal logic, the model may consist of a family of possible worlds. In computer applications, possible worlds are represented by sets of propositions that are true (facts) or necessarily true (laws).

Mental Maps, Images, and Models

Observation by the neuroscientist Antonio Damasio (2010):

“The distinctive feature of brains such as the one we own is their uncanny ability to create maps... But when brains make maps, they are also creating images, the main currency of our minds. Ultimately consciousness allows us to experience maps as images, to manipulate those images, and to apply reasoning to them.”

The maps and images form mental models of the real world or of the imaginary worlds in our hopes, fears, plans, and desires.

Words and phrases of language can be generated from them.

They provide a “model theoretic” semantics for language that uses perception and action for testing models against reality.

Like Tarski’s models, they define the criteria for truth, but they are flexible, dynamic, and situated in the daily drama of life.

2. Situation Semantics

Meaning depends on the context of the discourse, the situation, and the intentions of the participants.

But what is a situation?

- **A situation is an actual, hypothetical, or fictional region.**
- **For some reason, that region is significant for the participants.**
- **Language or logic may be used to describe a situation.**
- **Indexicals (pronouns and pointing words like *this* or *that*) relate the sentences to each other and to the situation.**

Problems of mapping language to logic:

- **How do we determine what situation(s) are significant for understanding a discourse or a document?**
- **How do we relate the indexicals to entities in the situations?**
- **How are situations related to background knowledge and to the intentions of the participants?**

Theories of Situations

Worlds and possible worlds are far too big:

- No human can comprehend or talk about an entire world.
- Perception, action, language, and thought are limited to situations.

Theories for AI reasoning systems by John McCarthy:

- Situation calculus (1963).
- Notes on formalizing context (1993).

Situation semantics and situation theory.

- Focus on natural language semantics by Barwise & Perry (1980, 1983).
- Later shift to a more abstract situation theory by Keith Devlin (1991).
- Studies of information flow by Barwise & Seligman (1997).
- Observations about the **unlimited effects of context** by Devlin (2005).
- The article by Devlin shows that there is no limit to the amount of context that might be relevant when the analysis is pursued in depth.
- Conclusion: Computers will not achieve or surpass human intelligence for a long, long time.

The Boundaries of a Situation

Definition: A situation is a real or imagined region of space-time that bounds the range of perception, action, interaction, and communication of one or more agents:

- The boundary of a situation is determined by the range of perception, action, and communication by the agents in it.
- A situation without agents is possible, but meaningless.
- Microscopes, telescopes, and TV use enhanced methods of perception and action to change the boundary of a situation.
- Psychologists and sociologists study human situations.
- Linguists and logicians formulate theoretical models of agents interacting in and talking about situations.
- Computer scientists develop methods for simulating and reasoning about the models.

Situation Theory

Based on a book by Barwise and Perry (1983) and developed mostly at Stanford (CSLI) during the 1980s and '90s.

In situation theory, the unit of information is called an *infor* σ , which is entailed by some *situation* s : $s \models \sigma$

The meaning of a language expression φ is a relation between a *discourse situation* d , a *speaker connection function* c , and a *described situation* e : $d, c \parallel \varphi \parallel e$

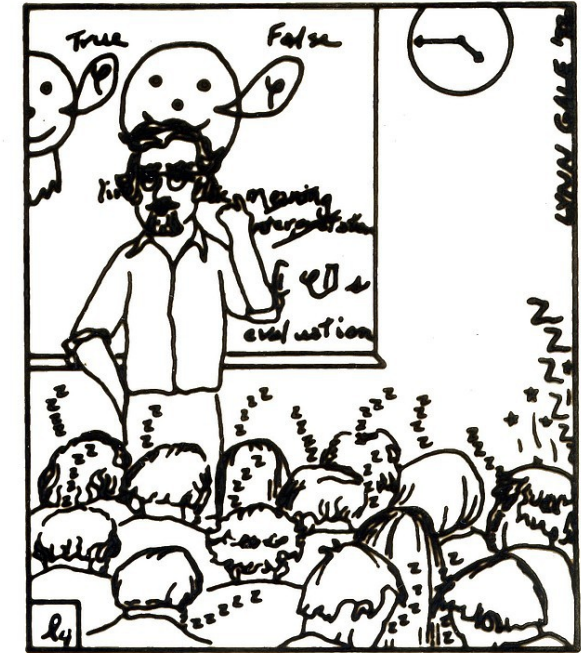
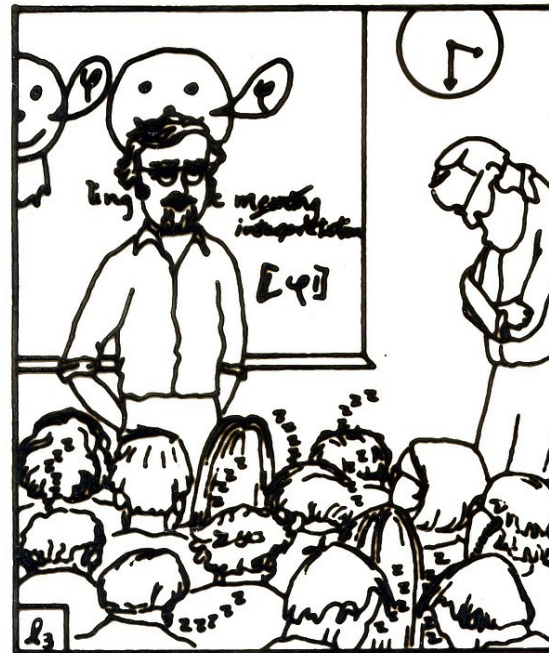
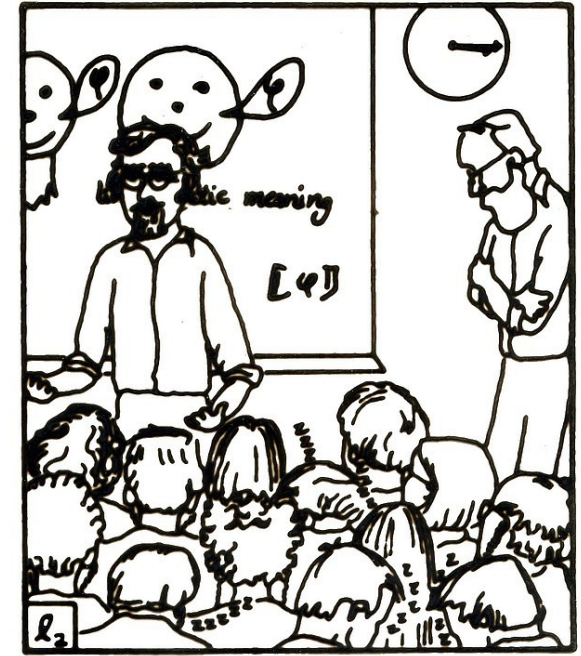
Those relations may be expressed in some versions of logic:

- An infor represents a relation and the things it relates.
- A compound infor is a Boolean combination of relations.
- But the logic would also require metalanguage or logic about logic.

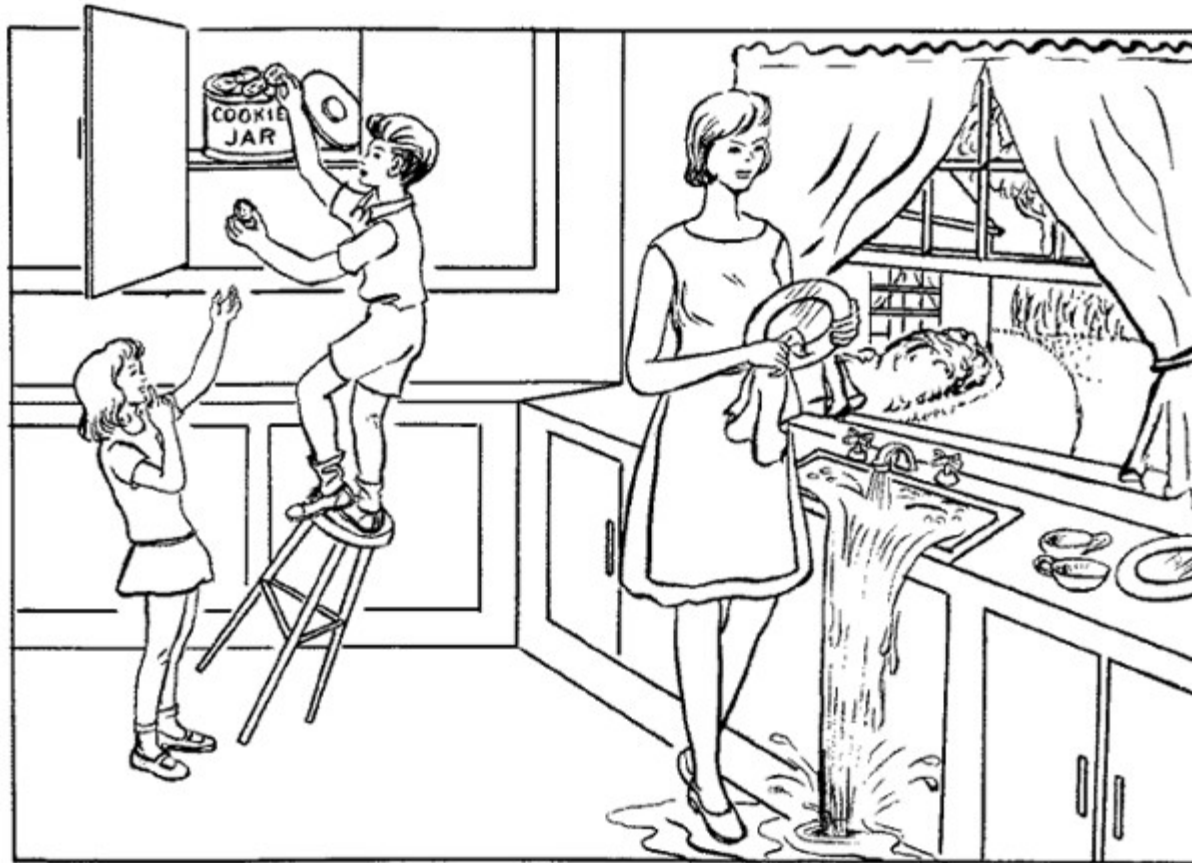
In the situation e ,
 John Perry is lecturing
 while Jon Barwise is
 standing on the right.

A language expression φ
 is a relation between a
discourse situation d , a
speaker connection
function c , and a
described situation e :
 $d, c \parallel \varphi \parallel e$.

If φ is the expression
 “the number of sleeping
 students”, its value is
 3 at 3:01 pm, 5 at 3:15,
 9 at 3:30, and 19 at 3:45.



Example of a Situation



This is a test picture used to diagnose patients with aphasia. A patient's description of the situation can show the effects of lesions caused by wound, stroke, tumor, or infection.

The "cookie theft" picture was adapted from Goodglass & Kaplan (1972).

Meaningful Aspects of the Situation

Space-time region of the “cookie theft” picture:

- Afternoon in the kitchen of a private home.

Agents:

- Girl, boy, woman.

Goals of the agents:

- Girl, boy: get cookies.
- Woman: wash dishes; maintain discipline.

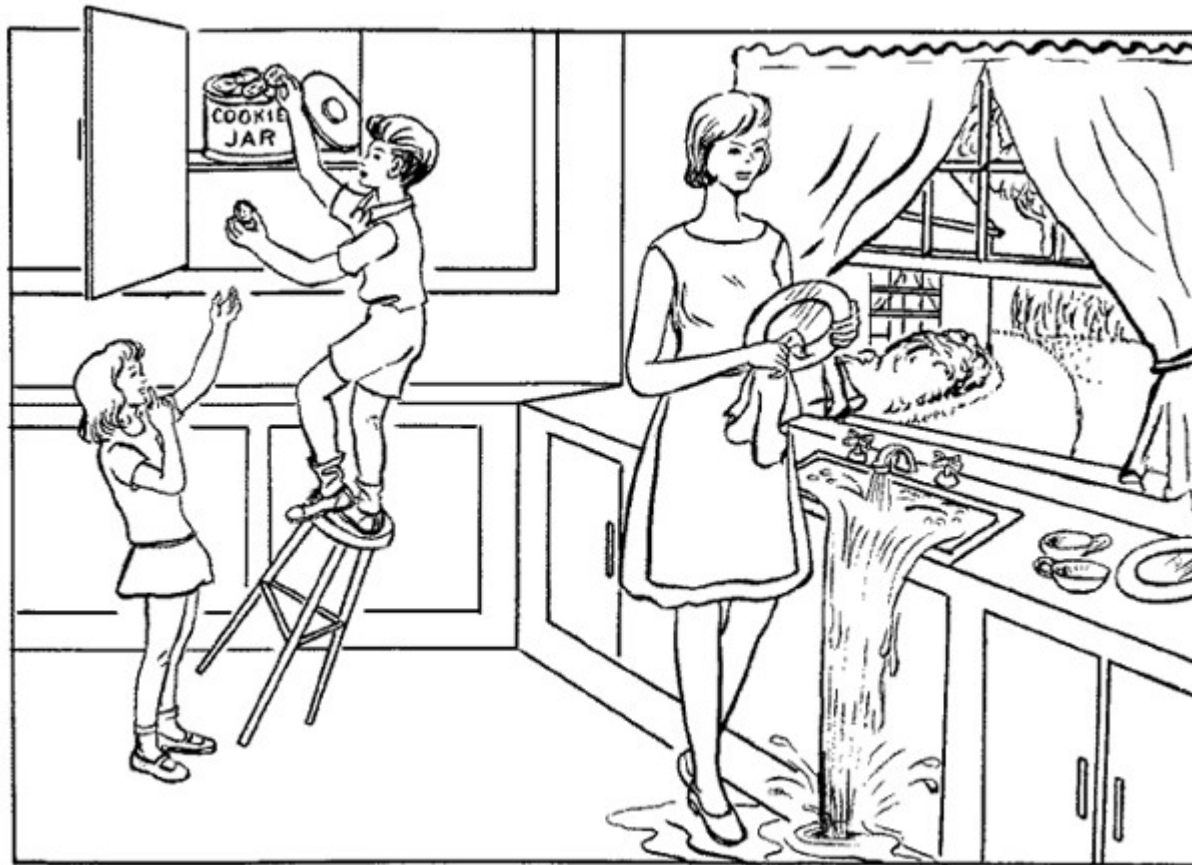
Actions:

- Wiping, spilling, reaching, holding, grasping, tipping, falling.

Question:

- How can we represent this situation in logic?

A Description in Controlled English



{Situation: A woman, a girl, and a boy are in a kitchen of a house. The woman wipes a plate with a cloth. Water spills on the floor of the kitchen. The girl reaches for a cookie. The boy holds a cookie in his left hand. The boy grasps a cookie with his right hand. The boy stands on a stool. The stool tips over. The boy falls down.}

What is a Situation?

Fundamental problem: Nobody could define it.

- **Barwise and Perry (1983): “A region of space-time.”**
“Thus we assume that the spatiotemporal relations are extensional and that there is a fixed fact about the matter about which relations hold of which locations.”
- **Keith Devlin (1991): Situations “include, but are not equal to any of, simply-connected regions of space-time, highly disconnected space-time regions, contexts of utterance..., collections of background conditions for a constraint, and so on.”**

Original goal: Define propositional attitudes (intentions) in terms of objective facts about clearly delimited regions.

Inevitable conclusion: The scope of a situation depends on the intentions of the participants in it. That fact defeats the hopes of deriving propositional attitudes from “objective facts”.

An Actual Conversation

A transcript, with background knowledge in italics: *

Husband: Dana succeeded in putting a penny in a parking meter today without being picked up.

This afternoon as I was bringing Dana, our four-year-old son, home from the nursery school, he succeeded in reaching high enough to put a penny in a parking meter when we parked in a meter zone, whereas before he has always had to be picked up to reach that high.

Wife: Did you take him to the record store?

Since he put a penny in a meter that means that you stopped while he was with you. I know that you stopped at the record store either on the way to get him or on the way back. Was it on the way back, so that he was with you or did you stop there on the way to get him and somewhere else on the way back?

Husband: No, to the shoe repair shop.

No, I stopped at the record store on the way to get him and stopped at the shoe repair shop on the way home when he was with me.

* From page 9 of [Confronting context effects](#), by Keith Devlin (2005).

The Context of a Conversation

The continuation of the previous conversation:

Wife: What for?

I know of one reason why you might have stopped at the shoe repair shop. Why did you in fact?

Husband: I got some new shoe laces for my shoes.

As you will remember I broke a shoe lace on one of my brown Oxfords the other day so I stopped to get some new laces.

Wife: Your loafers need new heels badly.

Something else you could have gotten that I was thinking of. You could have taken in your black loafers which need heels badly. You'd better get them taken care of pretty soon.

Observation by Devlin: *“The task [of specifying all the relevant context] was endless. At every stage, what has been specified is dependent on further contextual factors.”*

3. Representing Contexts in Logic

Sections 1 and 2 showed that a complete statement of all contextual information is difficult, even for humans.

But any contextual information that humans discover and state explicitly can be translated to logic.

Two operators are necessary and sufficient:

1. A relation that says proposition p is true in context c .
2. A metalevel operator that encapsulates a block of statements.

For #1, John McCarthy (1993) proposed the relation $ist(c,p)$.

For #2, the IKL logic has an operator named *that*.

Peirce and Tarski developed representations to distinguish the object language from metalanguage about the object language.

IKL is closer to Peirce's methods than to Tarski's methods.

Importance of Visualization

Observation by Jon Barwise and John Etchemendy about teaching logic at Stanford University: *

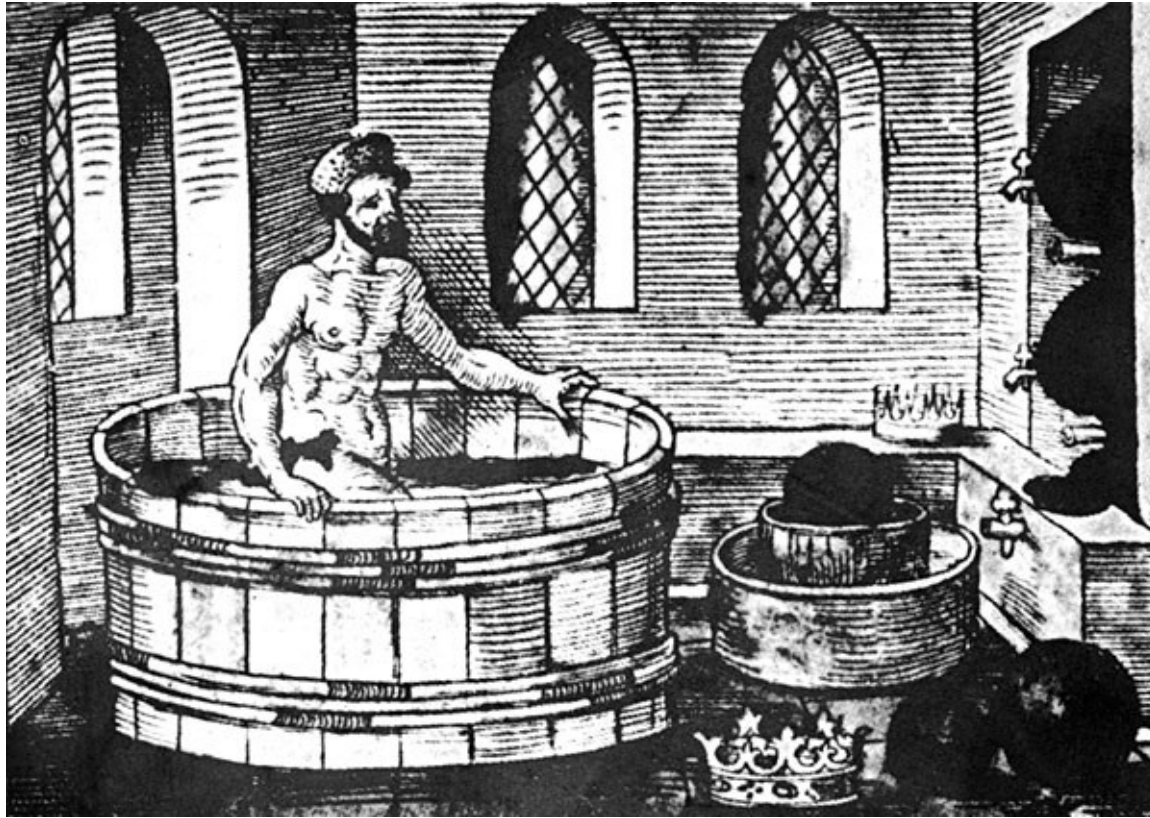
In our classes... students would make egregious errors in translating between sentences of English and sentences of first-order logic, errors that would have been inconceivable had they really understood the meanings of both sentences.

We were reminded, too, that over the years a handful of logicians, most notably Euler, Venn, and Peirce, had stressed the importance and interest of nonsentential inference. The diagrams of Euler and Venn, both of which use circles to represent collections of objects, are still widely known and used, even though their expressive power is sorely limited. C. S. Peirce, inspired by the utility of molecular diagrams in reasoning about chemical compounds, developed a more intricate and powerful diagrammatic formalism. While Peirce's system has not won over many human users, it has become an important tool in computer science.

Visualization is essential for creative insights in math, logic, and science.

* J. Barwise & J. Etchemendy, *Computers, visualization, and the nature of reasoning*, <http://kryten.mm.rpi.edu/COURSES/LOGAIS02/bar.etch.reasoning.pdf>

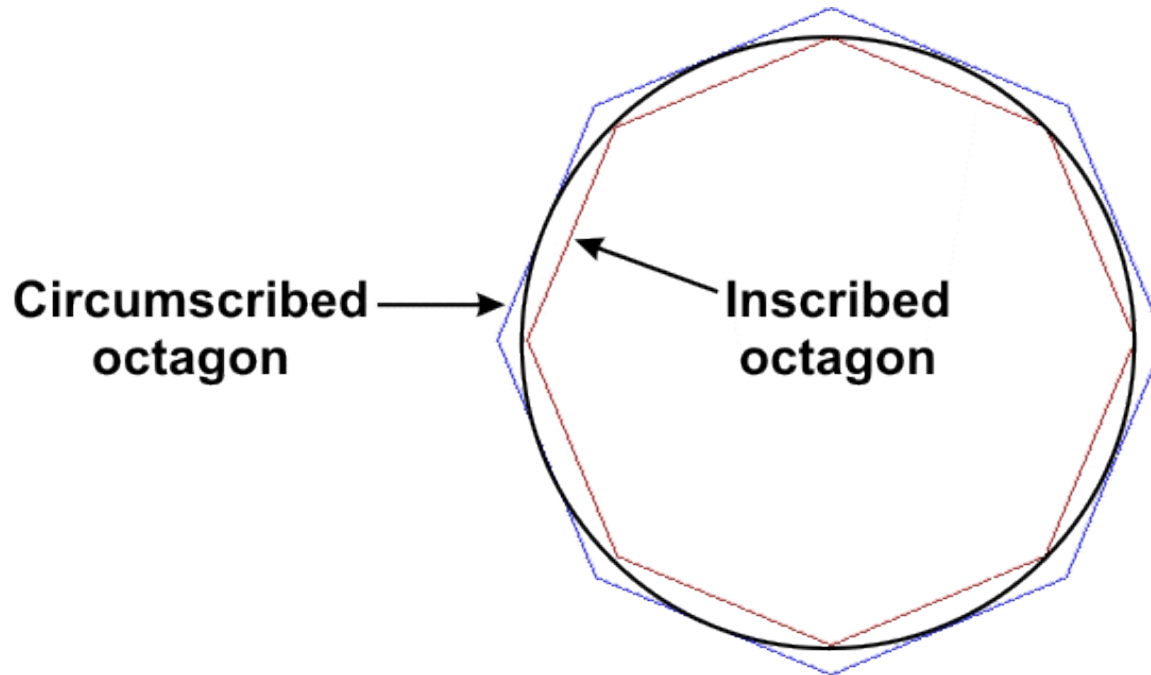
Archimedes' Eureka Moment



Insight: A submerged body displaces an equal volume of water.

- It's a mathematical principle, a property of Euclidean space.
- Scientists and engineers have used it ever since.
- They don't prove it. They use it to define *incompressible fluid*.

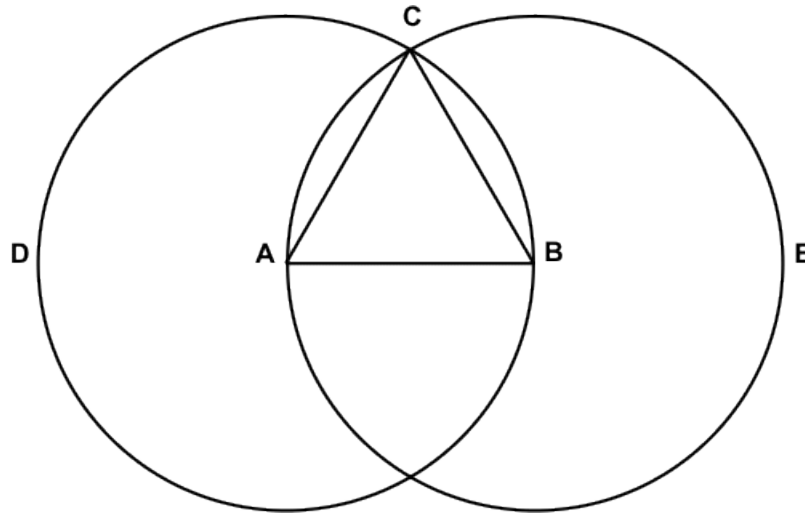
Determining the Value of π



Archimedes had two creative insights:

- The circumference of the circle is greater than the perimeter of the inner polygon and less than that of the outer polygon.
- As the number of sides increases, the inner polygon expands, and the outer polygon shrinks. They converge to the circle.
- Given these insights, a good mathematician could compute π to any desired precision. Archimedes used 96-gons.

Euclid's Proposition 1



Euclid's statement, as translated by Thomas Heath:

- **On a given finite straight line, to draw an equilateral triangle.**

The creative insight is to draw two circles:

- **The circle with center at A has radii AB and AC.**
- **The circle with center at B has radii BA and BC.**
- **Since all radii of a circle have the same length, the three lines AB, AC, and BC form an equilateral triangle.**

Contexts in Conceptual Graphs

Conceptual graphs were designed as a combination of Peirce's existential graphs with additional features from other logics. *

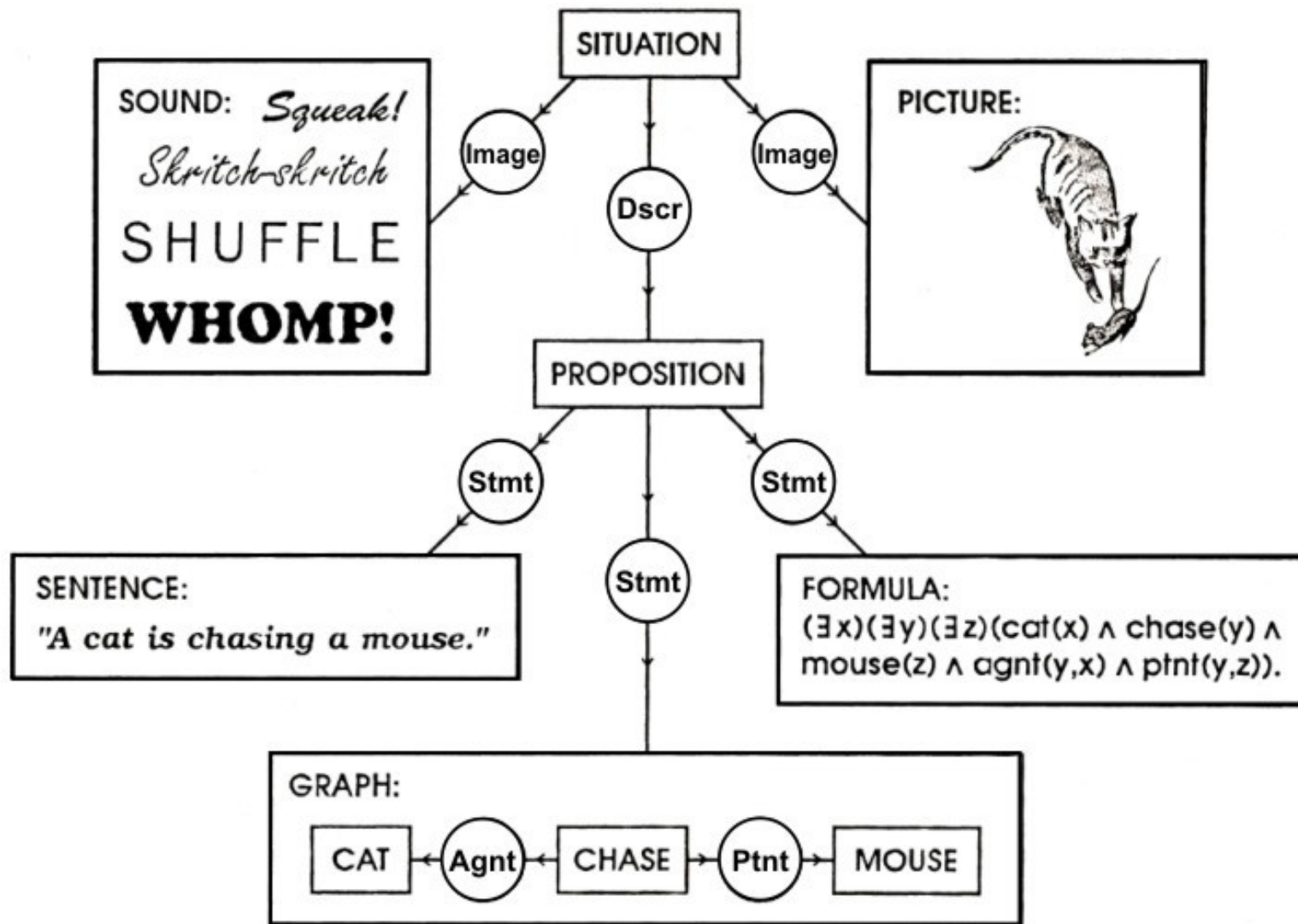
Metalanguage is represented by an enclosure, and its semantics is based on methods by Peirce, Tarski, and Hintikka.

When the ISO standard for Common Logic was developed, the Conceptual Graph Interchange Format (CGIF) adopted the formal semantics of Common Logic.

When IKL was developed, CGIF was extended to include the semantic extensions of IKL. **

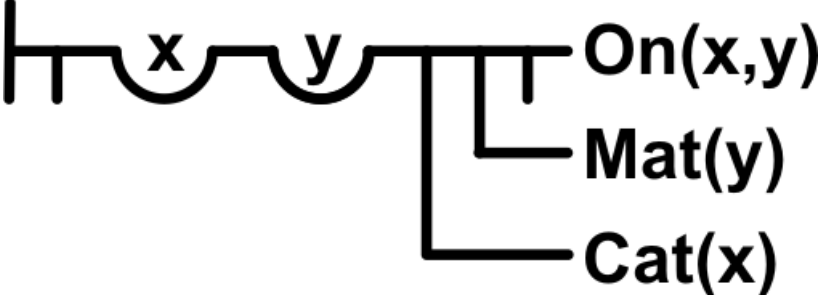
* See <http://jfsowa.com/pubs/eg2cg.pdf>

** See <http://jfsowa.com/ikl>



The boxes of a conceptual graph may contain images of the concept type. This CG states that some situation has a sound image and a picture image. The situation has a description (Dscr) by a proposition, which has a statement (Stmt) as an English sentence, a formula in predicate calculus, and a conceptual graph.

How to say “A cat is on a mat.”

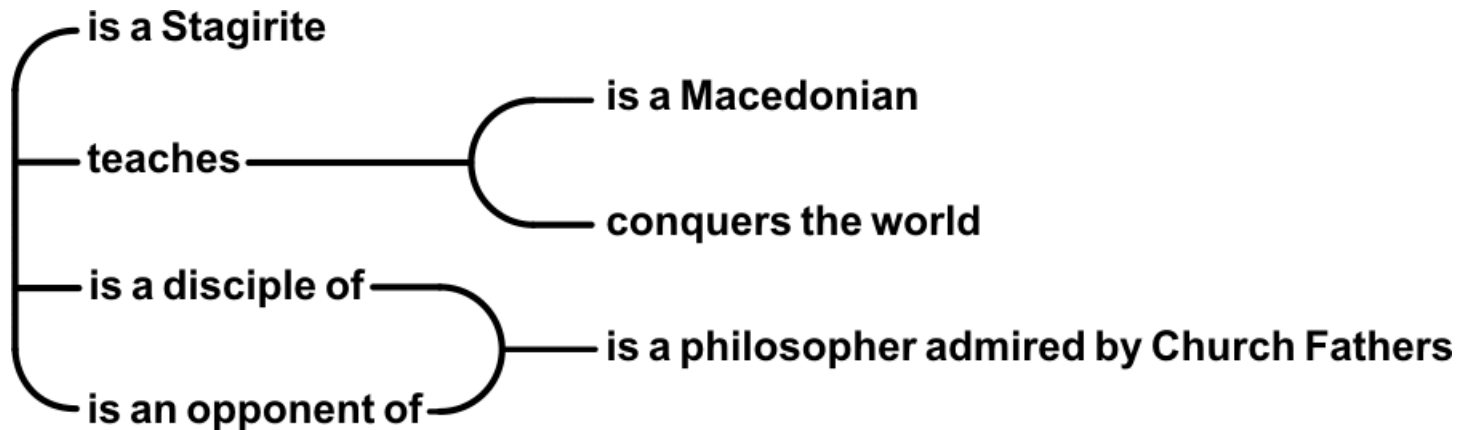
Gottlob Frege (1879):  The diagram shows a horizontal line with two small vertical bars on the left side. Below the line, there are two curved lines, one labeled 'x' and one labeled 'y'. From the right end of the horizontal line, a vertical line descends and then branches into three horizontal lines, each labeled with a predicate: 'On(x,y)', 'Mat(y)', and 'Cat(x)'.

Charles Sanders Peirce (1885): $\Sigma_x \Sigma_y \text{Cat}_x \cdot \text{Mat}_y \cdot \text{On}_{x,y}$

Giuseppe Peano (1895): $\exists x \exists y \text{Cat}(x) \wedge \text{Mat}(y) \wedge \text{On}(x,y)$

Charles Sanders Peirce (1897): **Cat — On — Mat**

Expressing Simple Relations



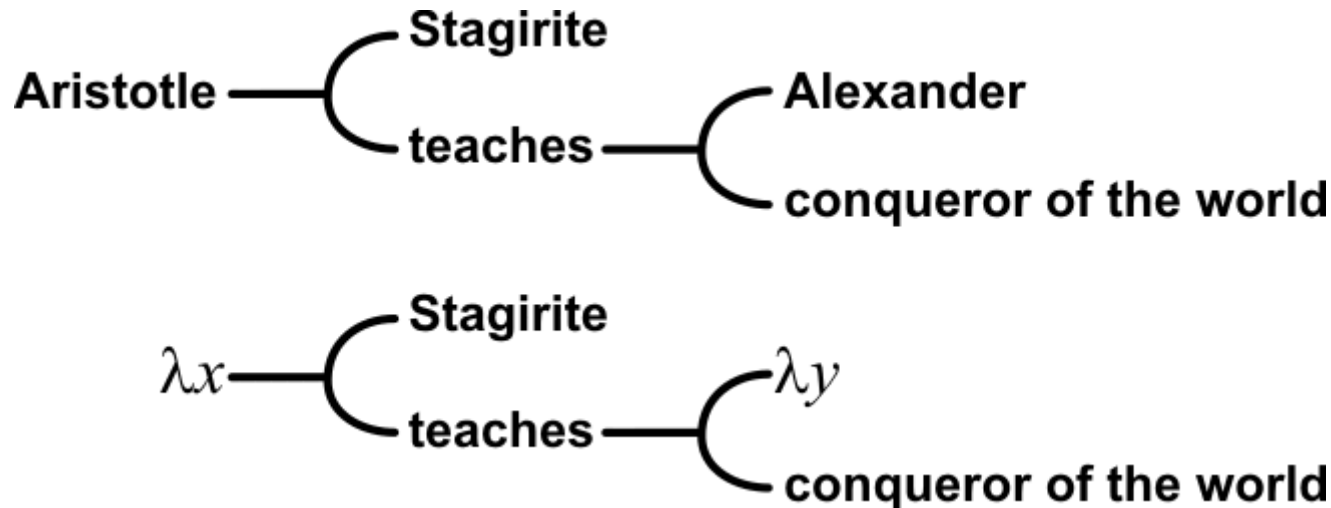
English: *“There exists an x , which is a Stagirite, x teaches some y , which is a Macedonian that conquers the world, x is a disciple of some z , which is a philosopher admired by Church Fathers, and x is an opponent of z .”*

The only logical operators are existence and conjunction:

$\exists x \exists y \exists z$ (isaStagirite(x) \wedge teaches(x,y) \wedge isaMacedonian(y) \wedge conquersTheWorld(y) \wedge isaDiscipleOf(x,z) \wedge isanOpponentOf(x,z) \wedge isaPhilosopherAdmiredByChurchFathers(z)).

This subset of logic can represent the content of any relational DB or any RDF DB (including blank nodes in RDF).

Lambda Abstraction



The top EG says *Aristotle is a Stagirite who teaches Alexander who conquers the world.*

In the EG below it, the names Aristotle and Alexander are erased, and their places are marked with the Greek letter λ .

That EG represents a dyadic relation: *is a Stagirite who teaches* *who conquers the world.*

Peirce used an underscore to mark those empty places, but Alonzo Church marked them with λ .

Syntax of Existential Graphs

Example: Cat — On — Mat

- Two lines mean there exists something x and something y .
- Cat and Mat are *monadic* relations; On is a *dyadic* relation.

Four primitives:

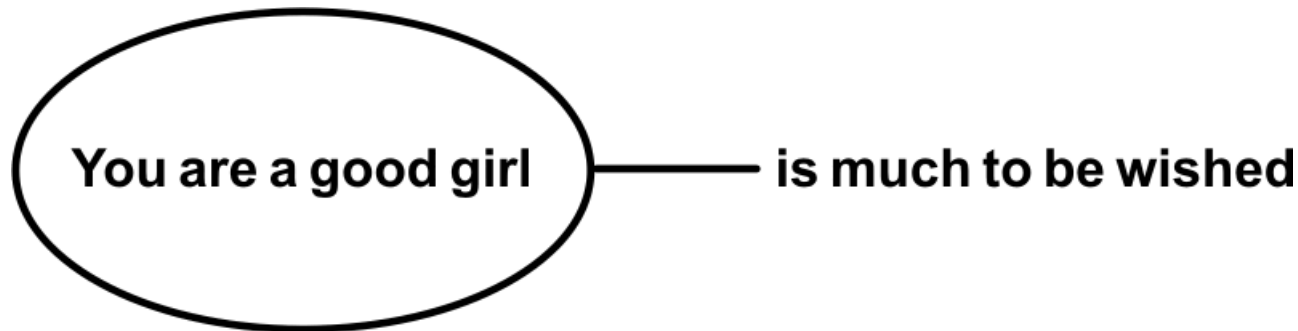
- Relation: A character string with zero or more *pegs*.
- Existence: A *line* that means “There exists something.”
- Conjunction: Two or more graphs in the same area.
- Metalanguage: An *oval* that encloses some graph or subgraph.

Four combinations:

- Argument: A line attached to a peg of some relation.
- Equality: Two or more connected lines (called a *ligature*).
- Metacomment: A line that connects an oval to a relation.
- Negation: A *shaded oval* that represents the comment *not*.

Peirce's Metalanguage

In 1898, C. S. Peirce extended first-order logic with a graphical enclosure for encapsulating one or more statements in logic: *



The relation named 'You are a good girl' has zero arguments. It represents an existential graph that states a proposition p .

The relation named 'is much to be wished' is attached to a line that states the existence of something that is wished.

With these features, Peirce's graphs could represent contexts and the operators for representing and reasoning about them.

* From Charles Sanders Peirce, *Reasoning and the Logic of Things*, The Cambridge Conferences Lectures of 1898, Harvard University Press, p. 151.

Tarski's Metalanguage

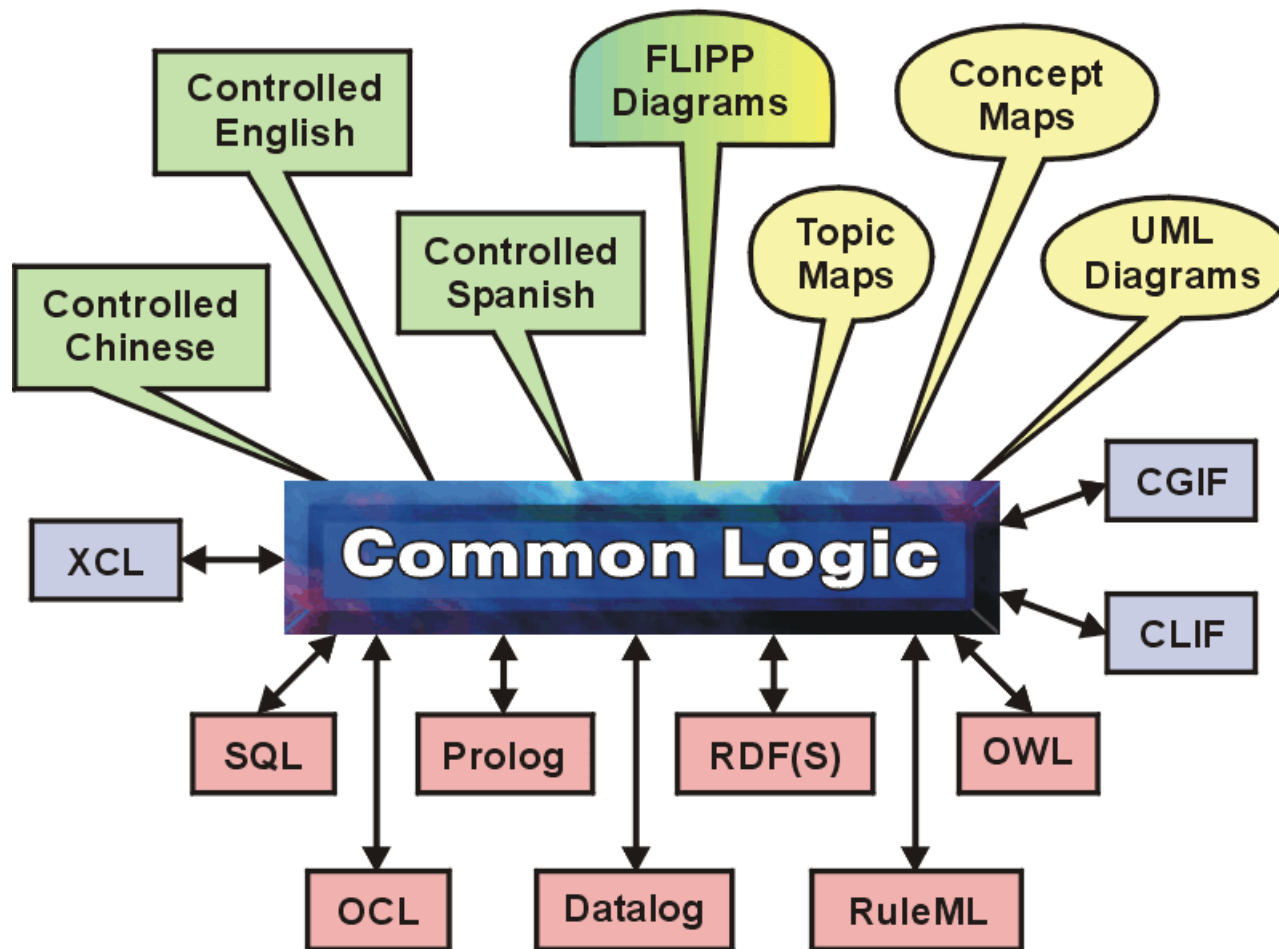
In his paper “The concept of truth in formalized languages,” Tarski (1933) used a metalanguage to specify truth in an object language.

For simplicity, he used the same syntax and semantics (first-order logic) for both the metalanguage and the object language

To avoid contradiction, Tarski kept the two levels distinct:

- The object level had a domain D , which included everything that the variables could refer to.
- But the metalanguage had a larger domain: it included D and all the syntactic features of the object language.
- Tarski also extended this principle to a hierarchy of metalanguages: the domain of each one included the domain of its object language plus all its syntactic features.

Human Interfaces



Machine Interfaces

Common Logic Controlled English

A dialect of Common Logic that looks like English.

CLCE uses a subset of English syntax and vocabulary.

But CLCE grammar avoids constructions that could be ambiguous.

CLCE replaces pronouns with temporary names called *variables*.

Examples:

For every company C ,
exactly one manager in C is the CEO of C ;
every employee of C except the CEO reports to the CEO;
the CEO of C does not report to any employee of C .

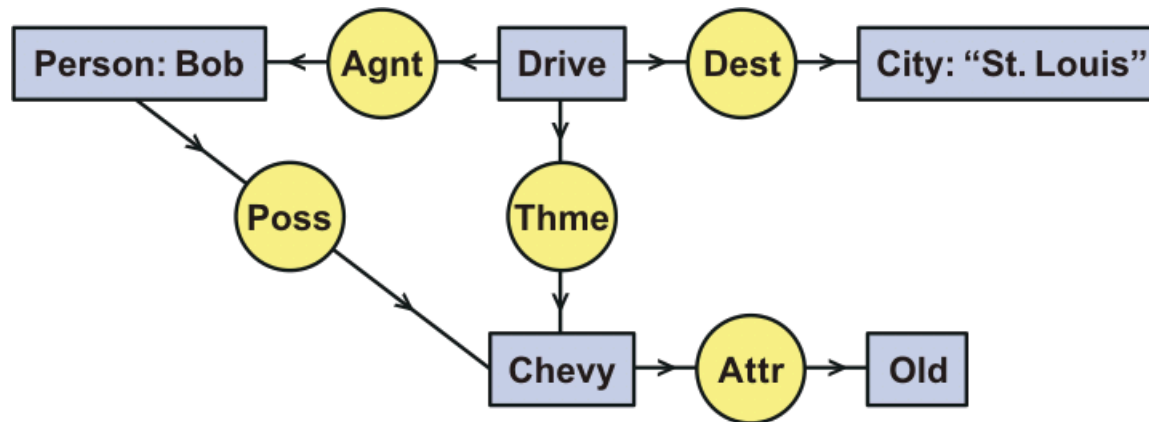
If an integer N is 5, then $(N^3 = 125)$.

The scope of variables, such as C or N , extends to the ending period.

Note: CLCE is not an ISO standard, but it uses the CL semantics.

CLCE: Bob drives his old Chevy to St. Louis.

Conceptual graph display form:



Conceptual Graph Interchange Format (CGIF):

```
[Drive *x] [Person Bob] [City "St. Louis"] [Chevy *y] [Old *z]
(Agnt ?x Bob) (Dest ?x "St. Louis") (Thme ?x ?y) (Poss Bob ?y)
(Attr ?y ?z)
```

Common Logic Interchange Format (CLIF):

```
(exists ((x Drive) (y Chevy) (z Old))
  (and (Person Bob) (City "St. Louis") (Agnt x Bob)
    (Dest x "St. Louis") (Thme x y) (Poss Bob y) (Attr y z)))
```

Representing Situations

Common Logic in any dialect – CLIF, CGIF, or CLCE – can represent a situation and the things and events in it.

But an extension to Common Logic is necessary to express theories about propositions and situations.

The critical extension is the ability to make statements about propositions and the situations they describe.

That extension makes it possible to talk about the goals of the people or other agents in the situation.

It also enables plans, hypotheses, reasoning, predictions, and evaluations about situations and their outcomes.

IKRIS Project

DoD-sponsored project: Design an Interoperable Knowledge Language (IKL) as an extension to Common Logic.

Goals:

- **Enable interoperability among advanced reasoning systems.**
- **Test that capability on highly expressive AI languages.**

Show that semantics is preserved in round-trip mapping tests:

- **Cycorp: Cyc Language \rightarrow IKL \rightarrow CycL**
- **RPI / Booz-Allen: Multi-Sorted Logic \rightarrow IKL \rightarrow MSL**
- **Stanford/IBM/Battelle: KIF \rightarrow IKL \rightarrow KIF**
- **KIF \rightarrow IKL \rightarrow CycL \rightarrow IKL \rightarrow MSL \rightarrow IKL \rightarrow KIF**

Conclusion: “IKRIS protocols and translation technologies function as planned for the sample problems addressed.”

Interoperable Knowledge Representation for Intelligence Support (IKRIS), Evaluation Working Group Report, prepared by David A. Thurman, Alan R. Chappell, and Chris Welty, Mitre Public Release Case #07-1111, 2007. See <http://www.jfsowa.com/ikl/>

The IKL Extension to Common Logic

Common Logic is a superset of the logics used in many semantic systems, but some systems require even more expressive logics.

Only one new operator is needed: a metalanguage enclosure, which uses the keyword 'that' to mark the enclosed statement.

- **The enclosed statement denotes a proposition.**
- **That proposition could be a conjunction of many statements.**
- **It can be given a name, and other propositions can refer to it.**
- **In effect, IKL can be used as a metalanguage for talking about and relating packages of IKL statements nested to any depth.**

CL with the IKL extensions can represent a wide range of logics for modality, defaults, probability, uncertainty, and fuzziness.

For the IKL extension, see <http://www.ihmc.us/users/phayes/IKL/SPEC/SPEC.html>
and <http://www.ihmc.us/users/phayes/ikl/guide/guide.html>

Using CLCE to Express IKL

The operator 'that' of IKL can be used in CLCE:

Tom believes that Mary knows that $(2 + 2 = 4)$.

In CLIF notation for IKL:

(Believes Tom (that (Knows Mary (that (= (+ 2 2) 4))))))

In CGIF notation for IKL:

(Believes Tom [Proposition (Knows Mary [Proposition (+ 2 2 | 4)])])

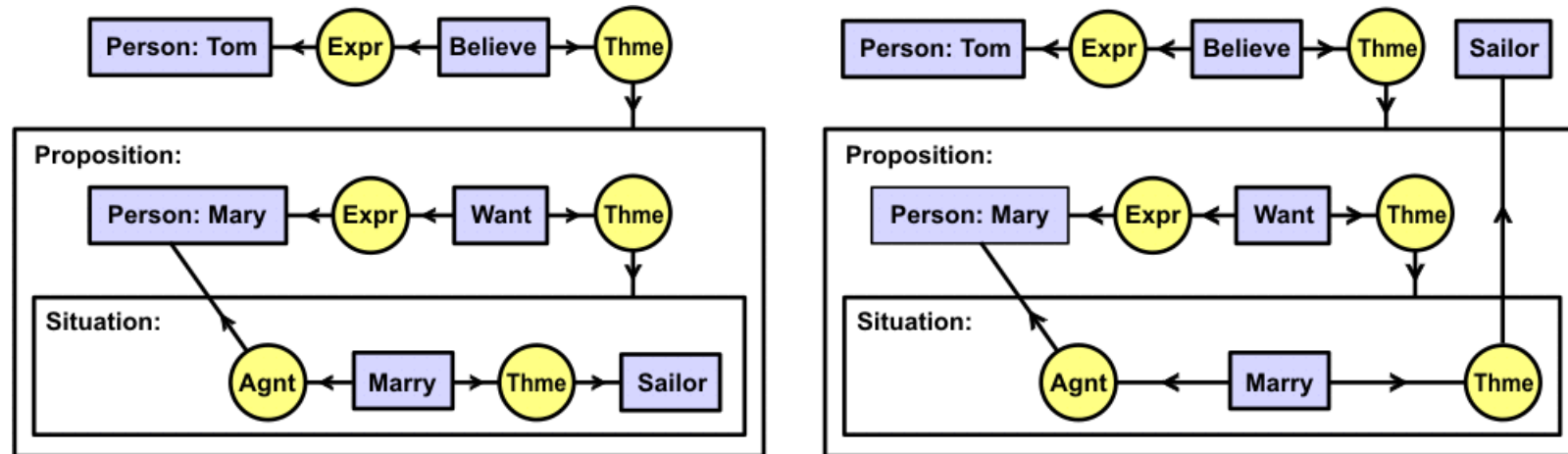
The operator 'that' is a powerful metalevel extension.

It enables IKL to specify languages, define their semantics, and specify transformations from one language to another.

Writing complex statements in CLCE requires training in logic.

But anybody who can read English can read CLCE.

Propositions and Situations



The two CGs above show two different interpretations of the English sentence *Tom believes that Mary wants to marry a sailor*:

- *There exists a sailor, and Tom believes a proposition that Mary wants a situation in which she marries the sailor.*
- *Tom believes a proposition that Mary wants a situation in which there exists a sailor whom she marries.*

A situation is a meaningful region of space-time described by the proposition stated by the nested CG.

Representing IKL in CLIF and CGIF

Following is the CGIF representation for the CG on the left of the previous slide:

```
[Person: Tom] [Believe: *x1] (Expr ?x1 Tom) (Thme ?x1 [Proposition:  
  [Person: Mary] [Want: *x2] (Expr ?x2 Mary) (Thme ?x2 [Situation:  
    [Marry: *x3] [Sailor: *x4] (Agnt ?x3 Mary) (Thme ?x3 ?x4)]))])
```

In CLIF notation, the operator 'that' applied to a CL or IKL sentence denotes the proposition stated by the sentence:

```
(exists ((x1 Believe)) (and (Person Tom) (Expr x1 Tom) (Thme x1 (that  
  (exists ((x2 Want) (s Situation)) (and (Person Mary) (Expr x2 Mary)  
    (Thme x2 s) (Dscr s (that  
      (exists ((x3 Marry) (x4 Sailor)) (and (Agnt x3 Mary) (Thme x3 x4)  
        )))))))))))
```

To represent the CG on the right of the previous slide, move the concept node [Sailor: *x4] in front of the concept [Person: Tom] for CGIF notation. For CLIF, move (x4 Sailor) in front of (x1 Believe).

Integrating Logic and Imagery

Peirce called existential graphs “the logic of the future.”

Computer graphics and virtual reality can implement them:

- **The icons in two-dimensional maps can be generalized to three dimensions and even 4 dimensions for motion and change.**
- **Conjunctions and lines of identity can be represented in any dimension.**
- **For negation, the ovals can be generalized to closed shapes in any number of dimensions. Rvachev functions can implement them.**
- **Viewers with VR goggles could wander through 4-dimensional EGs, watch the movies, and manipulate icons according to Peirce’s rules.**

Peirce’s claim is consistent with neuroscience: *

- **As Damasio said, images are “the main currency of our minds.”**
- **As Johnson-Laird observed, Peirce’s rules of inference insert and erase graphs and subgraphs — operations that neural processes can perform.**
- **Generalized EGs can include arbitrary images in the graphs.**
- **When Peirce claimed that EGs represent “a moving picture of the action of the mind in thought,” he may have imagined something similar.**

* See “Peirce, Polya, and Euclid”: <http://jfsowa.com/talks/ppe.pdf>

Related Readings

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<http://www.helsinki.fi/~pietarin/publications/Semiotica-Diagrams-Pietarinen.pdf>

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<http://www.jfsowa.com/pubs/rolelog.pdf>

Sowa, John F. (2006) Peirce's contributions to the 21st Century,
<http://www.jfsowa.com/pubs/csp21st.pdf>

Documents and slides about IKL and related projects, <http://www.jfsowa.com/ikl/>

For other references, see the general bibliography,
<http://www.jfsowa.com/bib.htm>