Five Questions on Epistemic Logic

John F. Sowa

This is a preprint of Chapter 23 in *Epistemic Logic: 5 Questions*, edited by Vincent F. Hendricks & Olivier Roy, Automatic Press, New York, 2010, pp. 231-241.

1. Why were you initially drawn to epistemic logic?

My work on epistemic logic developed from my research in artificial intelligence. My formal education in mathematics and my interests in philosophy and linguistics led me to Richard Montague's synthesis of a formal grammar with an intensional semantics based on Kripke's possible worlds. Around the same time, however, my readings in psycholinguistics showed that young children learn to use modal expressions with far greater ease and flexibility than graduate students learn Montague's logic. Following are some sentences spoken by a child named Laura who was less than three years old (Limber 1973):

> Here's a seat. It must be mine if it's a little one.
>
> I want this doll because she's big.
>
> When I was a little girl I could go "geek-geek" like that. But now I can go "this is a chair."

In these sentences, Laura correctly expressed possibility, necessity, tenses, indexicals, conditionals, causality, quotations, and metalanguage about her own language at different stages of life. She already had a fluent command of a much larger "fragment of English" than Montague had formalized.

The syntactic theories by Chomsky and his followers were far more detailed than Montague's, but their work on semantics and pragmatics was rudimentary. Some computational linguists had developed a better synthesis of syntax, semantics, and pragmatics, but none of their programs could interpret, generate, or learn language with the ease and flexibility of a child. The semantic networks of artificial intelligence provided a flexible computational framework with a smooth mapping to and from natural languages. In 1976, I published my first paper on conceptual graphs as a computable notation for natural language semantics, but I hoped to find a simpler and more natural representation for the logical operators and rules of inference.

In 1978, I discovered Peirce's existential graphs. Peirce had developed three radically different notations for logic: the relational algebra, the algebraic notation for predicate calculus, and the existential graphs, which he developed in detail during the last two decades of his life. Although his original existential graphs were limited to first-order logic, Peirce added further innovations for representing modality, metalanguage, and higher-order logic (Roberts 1973). Other logicians have since developed these topics in greater detail, but often in isolation from broader issues of science and philosophy. Peirce, however, sought to integrate every aspect of semiotics by developing his graphs as a "system for diagrammatizing intellectual cognition" (MS 292:41). My studies in AI, linguistics, and philosophy convinced me that such a synthesis is a prerequisite for understanding human intelligence or developing an adequate computer simulation.

2. What examples from your work, or work of others, illustrate the relevance of epistemic logic?

Epistemic logic belongs to the tradition from Aristotle to Peirce of using logic to analyze science, language, thought, and reasoning. Unlike Frege and Russell, who focused on the foundations of mathematics, Peirce integrated logic with semiotics and applied it to every area of science, philosophy, and language. To represent modal contexts in existential graphs, Peirce experimented with colors and dotted lines. Frege insisted on a single domain of quantification called *everything*, but Peirce used *tinctures* to distinguish different *universes*:

> The nature of the universe or universes of discourse (for several may be referred to in a single assertion in the rather unusual cases in which such precision is required) is denoted either by using modifications of the heraldic tinctures, marked in something like the usual manner in pale ink upon the surface, or by scribing the graphs in colored inks. (MS 670:18)

Peirce considered three universes: actualities, possibilities, and the necessitated. He subdivided each universe in four ways to define 12 modes. In the universe of possibilities, for example, he distinguished objective possibility (an alethic mode), subjective possibility (epistemic), social possibility (deontic), and an interrogative mode, which corresponds to scientific inquiry by hypothesis and experiment. For the necessitated, he called the four subdivisions the rationally necessitated, the compelled, the commanded, and the determined. Most of his writings on these topics were unpublished, and he changed his terminology from one manuscript to the next. Peirce admitted that a complete analysis and classification would be "a labor for generations of analysts, not for one" (MS 478:165).

During the 20th century, Peirce's broad approach influenced logicians and philosophers who developed theories of epistemology. The first was Clarence Irving Lewis, whose teachers at Harvard included the pragmatists William James and Josiah Royce. After earning his PhD in 1910, Lewis lectured at Berkeley, where he wrote his *Survey of Symbolic Logic* in 1918. In the historical introduction, he devoted 21 pages to Boole, 28 pages to Peirce, and 11 pages to Whitehead and Russell. Most of the expository material covered classical first-order logic, but Lewis devoted 49 pages to his own research on strict implication, which eventually led to his pioneering work on modal logic. In 1920, he joined the Harvard faculty, where he spent two years studying the manuscripts donated by Peirce's widow. After those studies, Lewis began to call himself a *conceptual pragmatist*, and he continued to write and lecture on modal logic, epistemology, and related topics for the rest of his life (Murphey 2005).

Other logicians with interests in epistemology and modal logic were also strongly influenced by Peirce. Arthur Prior quoted Peirce extensively in his writings on propositions (1976) and temporal logic (1977). In a 16-page critique of Wittgenstein's *Tractatus*, Frank Ramsey noted that Peirce's type-token distinction could clarify an ambiguity in Wittgenstein's analysis of propositions. In his later discussions with Wittgenstein, Ramsey recommended Peirce's writings (Nubiola 1996). Wittgenstein apparently accepted that advice because he recommended some papers by Peirce (1923) in a letter to his sister. Jaakko Hintikka has also written about both Peirce and Wittgenstein. Like Peirce, these philosophers applied logic to a broader range of issues than just the foundations of mathematics.

3. What is the proper role of epistemic logic in relation to other disciplines?

As a version of modal logic for reasoning about knowledge and belief, epistemic logic is intimately related to other modal logics and to the methods for acquiring, learning, and reasoning about knowledge. Every issue related to those topics is directly or indirectly relevant to epistemic logic; e.g., the criticisms by Dana Scott (1973):

> Formal methods should only be applied when the subject is ready for them, when conceptual clarification is sufficiently advanced... No modal logician really knows what he is talking about in the same sense that we know what mathematical entities are. This is not to say that the work to date in modal logic is all bad or wrong, but I feel that insufficient consideration has been given to questioning appropriateness of results... it is all too tempting to refine methods well beyond the level of applicability.

C. I. Lewis (1960) made a similar observation:

> It is so easy... to get impressive "results" by replacing the vaguer concepts which convey real meaning by virtue of common usage by pseudo precise concepts which are manipulable by "exact" methods — the trouble being that nobody any longer knows whether anything actual or of practical import is being discussed.

Quine (1972) noted that Kripke models can be used to prove that modal axioms are consistent, but they don't explain the intended meaning of those axioms:

> The notion of possible world did indeed contribute to the semantics of modal logic, and it behooves us to recognize the nature of its contribution: it led to Kripke's precocious and significant theory of models of modal logic. Models afford consistency proofs; also they have heuristic value; but they do not constitute explication. Models, however clear they be in themselves, may leave us at a loss for the primary, intended interpretation.

Kripke's major achievement was to demonstrate the consistency of modal logic and the axioms by C. I. Lewis in terms of set theory. But he assumed two primitives: a set of undefined entities called *possible worlds* and an undefined relation R among worlds. If R(w_1,w_2) happens to be true, then the world w_2 is said to be *accessible* from the world w_1. Kripke made some informal remarks to justify those primitives, but Scott and Quine would agree with Lewis: "nobody any longer knows whether anything actual or of practical import is being discussed."

A major limitation of most modal axioms and Kripke models is that they only relate a single modality. They permit dubious combinations such as *possibly necessary*, but they can't represent the sentence *I know that I'm never obligated to do anything impossible*, which combines epistemic, temporal, deontic, and alethic modes. Scott (1970) considered the limitation to just one modality "one of the biggest mistakes of all in modal logic":

> The only way to have any philosophically significant results in deontic or epistemic logic is to combine these operators with: Tense operators (otherwise how can you formulate principles of change?); the logical operators (otherwise how can you compare the relative with the absolute?); the operators like historical or physical necessity (otherwise how can you relate the agent to his environment?); and so on and so on.

To represent and reason about multiple modalities and their interactions, some logicians have added multiple accessibility relations to the Kripke models. Cohen and Levesque (1990), for example, introduced a *belief accessibility relation* for reasoning about knowledge and belief and a *goal accessibility relations* for desires and intentions. To justify the formalism, they related their axioms to the independently developed theory of beliefs, desires, and intentions by Bratman (1987). That justification is important, but Laura at age three could already use modal verbs to express her beliefs, desires, and intentions. The linguistic mechanisms that support her language are likely to be simpler than Kripke models with multiple accessibility relations.

Modern logicians have gone far beyond Peirce in developing axioms for various modalities, but they neglected his goal of classifying and relating the modalities in a systematic framework. Although Peirce never completed the framework, his four-way subdivision is still worth exploring:

1. Objective reality (actual, possible, or necessary) is independent of what any observer thinks. These are the alethic modes.

2. Each individual's subjective experience is a personal actuality in which beliefs develop, are verified, and become knowledge. These are the epistemic modes.

3. Each social group, ranging from a nuclear family to nation states, develops a consensus of habits, conventions, norms, regulations, and laws. These define the deontic modes.
4. Peirce regarded science as a collaboration among inquirers working on common interests and problems over a time span of millennia. At any point in time, their consensus about observations, hypotheses, and laws is a good, but fallible approximation to the objective modes.

This classification can be applied to any individual or social group at any stage of development. It can be used to analyze the way a child learns and uses modal terms. It can also be used to study social norms, legal systems, or scientific practice and methodology.

4. Which topics and contributions should have deserved more attention in late 20th century epistemic logic?

I have found the most fertile sources of new ideas for further research in historical writings that have been forgotten or in unpopular approaches that have been ignored. When I was doing the early research on conceptual graphs, I came across a collection of papers on "alternative semantics" edited by Hugues Leblanc (1973). Some of them, such as the one by Scott, contained stimulating advice and criticism. The two I used to define the semantics of CGs were by Michael Dunn and Jaakko Hintikka.

Dunn (1973) showed that each possible world in a Kripke model could be replaced by a pair of sets (M,L). For each world *w* in Kripke's sense, M is a Hintikka-style *model set* of all the facts that are true about *w*. Dunn's innovation is to designate L as the set of laws of *w*, which are necessarily true of *w*. For any Kripke model, there is always a Dunn model with an isomorphism of each world *w* to a pair (M,L) of the facts and laws of *w*. This isomorphism guarantees that any theorems derived in terms of Kripke models are also true of Dunn models. For that reason, most logicians have ignored Dunn models.

What makes Dunn models interesting is that the accessibility relation is no longer primitive. It can be derived from the choice of laws. Dunn defined accessibility from any world w_i to a world w_j to mean that the laws L_i of the first world are a subset of the facts M_j of the second world:

$$R(w_i,w_j) \equiv L_i \subset M_j.$$

According to this definition, the laws of the first world w_i are true in the second world w_j, but they may be demoted from the status of laws to merely contingent facts. Dunn's definition of accessibility holds for *normal models*, in which every law is also a fact, and for *non-normal models*, in which some laws might not be facts. In deontic logic, for example, the laws are obligatory, but sinners might violate them. For such models, the accessibility relation R is not reflexive because a non-normal world is not accessible from itself. This point is one more example of the nonintuitive nature of Kripke models: it seems odd to say that any world could be inaccessible from itself, especially the real world because it happens to have sinners. But there is nothing odd or counterintuitive about saying that some laws might be violated.

Although Dunn models and Kripke models are isomorphic when only one modality is considered, Dunn models are much easier to generalize for multimodal reasoning (Sowa 2003, 2006). With Kripke models, a new accessibility relation must be added as an undefined primitive for each type of modality. With Dunn models, each law can be tagged with a metalevel marker that indicates its source as a law of physics, chemical engineering, the United States, some local town, or some business policy. If no context is indicated, a general modal verb such as *must* would indicate necessity as determined by the conjunction of all applicable laws. For a particular context, a subscript could indicate that some subset of laws L_i determines the necessity operator $\Box_i$ and the possibility operator $\Diamond_i$. With this approach, no special primitives or assumptions are needed. Metalevel reasoning can be used to select the applicable laws for any particular problem, and the object-level reasoning would follow the usual first-order rules of inference.

The paper by Hintikka (1973) introduced two techniques that I adopted for defining the semantics of conceptual graphs and using them in processing natural language: *surface models* and *game theoretical semantics*. Hintikka's surface models anticipated the *dynamic semantics* of Groenendijk and Stokhof (1991). In both methods, the process of interpreting a sentence constructs a partial model of a world in which the sentence happens to be true. Hintikka's tree representation for surface models is easily adapted to graphs. His game-theoretical semantics happens to be closely related to Peirce's method of *endoporeutic* (outside-in evaluation) for evaluating the truth of an existential graph in terms of a model that could also be represented as a graph. The combination of these techniques provides a dynamic method for interpreting natural language text or dialog in graph models and evaluating the truth of logical graphs in terms of such models. It is also a realistic psycholinguistic hypothesis about the way people understand language, and it supports Peirce's claim that graphs can capture a "moving picture of the action of the mind in thought" (Pietarinen 2005).

5. What are the most important open problems in epistemic logic and what are the prospects for progress?

The most important task is to integrate the principles and methods discussed in the previous four answers: the relevance of epistemic logic to learning, using, and communicating knowledge; the psycholinguistic issues raised by child language and learning; the efficiency and flexibility of computer implementations; the empirical justification of the abstract formalisms; the classification of all modalities including the epistemic; the dynamic changes in knowledge and belief during a dialog; and the use of these principles in the design of intelligent agents that can communicate with people in ordinary language.

As Peirce observed, this is a task for "generations of analysts." But several generations have already passed, and we can use his guidelines to evaluate the progress. The first is Peirce's principle of pragmatism, which C. I. Lewis adopted: "The elements of every concept enter into logical thought at the gate of perception and make their exit at the gate of purposive action; and whatever cannot show its passports at both those two gates is to be arrested as unauthorized by reason" (CP 5.212). Epistemic logic, by itself, isn't directly connected to either gate. Lewis made some connections by alternating his work on abstract logic with his writings on epistemology. But a more complete solution must situate epistemic logic within a framework that includes both gates and all the interconnections.

No deductive logic can link all the steps from perception to action. Peirce emphasized the three methods of reasoning: induction, abduction, and deduction. The first two are necessary to derive the premises, from which deduction can derive the consequences. But those consequences must be tested against further observations, which lead to another round of induction, abduction, and deduction. Instead of an epistemic logic, the result is an epistemic cycle:

1. **Induction**. Perception supplies the observations from which induction derives generalizations. But memory to store previous observations and analogy to find similar observations are also required to gather the data for induction.
2. **Abduction**. For any particular purpose, abduction is required to gather and combine relevant observations and generalizations in novel hypotheses or to revise and refine previous hypotheses. The results are the premises used by deduction.

3. **Deduction**. Given a theory, deduction can derive the consequences, answer questions, or make predictions. The results may be stored in memory as a basis for further reasoning, but testing is necessary to ensure their accuracy and relevance.
4. **Action**. The results of deduction are a guide to actions upon the world. Those actions may lead to further observations, which may confirm, contradict, or refine the predictions. Then the cycle continues with a new round of inductions, abductions, deductions, and actions.

Note that deduction covers only 25% of the cycle from perception to action. The most important problem for epistemic logic is to explore the entire cycle and all its connections.

References

Bratman, Michael E. (1987) *Intentions, Plans, and Practical Reason*, Harvard University Press, Cambridge, MA.

Cohen, Philip R., & Hector J. Levesque (1990) Intention is choice with commitment, *Artificial Intelligence* **42:3**, 213-261.

Dunn, J. Michael (1973) A truth value semantics for modal logic, in Leblanc (1973) pp. 87-100.

Groenendijk, Jeroen, & Martin Stokhof (1991) Dynamic predicate logic, *Linguistics and Philosophy* **14:1**, 39-100.

Hintikka, Jaakko (1973) Surface semantics: definition and its motivation, in Leblanc (1973) pp. 128-147.

Leblanc, Hugues, ed. (1973) *Truth, Syntax and Modality*, North-Holland, Amsterdam.

Lewis, Clarence Irving (1918) *Survey of Symbolic Logic*, University of California Press, Berkeley.

Lewis, Clarence Irving (1960) Personal letter, quoted by Hao Wang (1986) *Beyond Analytic Philosophy: Doing Justice to What We Know*, MIT Press, Cambridge, MA, p. 116.

Limber, John (1973) The genesis of complex sentences, in T. Moore, ed., *Cognitive Development and the Acquisition of Language*, Academic Press, New York, 169-186.

Murphey, Murray G. (2005) *C. I. Lewis: The Last Great Pragmatist*, State University of New York, Albany.

Nubiola, Jaime (1996) Scholarship on the relations between Ludwig Wittgenstein and Charles S. Peirce, in I. Angelelli & M. Cerezo, eds., *Proceedings of the III Symposium on the History of Logic*, Gruyter, Berlin.

Peirce, Charles Sanders (1923) *Chance, Love and Logic: Philosophical Essays*, edited by Morris R. Cohen, Harcourt, Brace, & Co., New York.

Peirce, Charles Sanders (CP) *Collected Papers of C. S. Peirce*, ed. by C. Hartshorne, P. Weiss, & A. Burks, 8 vols., Harvard University Press, Cambridge, MA, 1931-1958.

Pietarinen, Ahti-Veikko (2005) Peirce's magic lantern of logic: Moving pictures of thought, *Transactions of the Charles S. Peirce Society.*

Prior, Arthur N. (1976) *The Doctrine of Propositions and Terms*, edited by P. T. Geach & A. J. P. Kenny, University of Massachusetts Press, Amherst.

Prior, Arthur N., & Kit Fine (1977) *Worlds, Times, and Selves*, University of Massachusetts Press, Amherst.

Roberts, Don D. (1973) *The Existential Graphs of Charles S. Peirce*, Mouton, The Hague.

Scott, Dana S. (1970) Advice on modal logic, in K. Lambert, ed., *Philosophical Problems in Logic*, D. Reidel, Dordrecht, pp. 143-173.

Scott, Dana S. (1973) Background to formalization, in Leblanc (1973) pp. 244-273.

Sowa, John F. (1976) Conceptual graphs for a database interface, *IBM Journal of Research and Development* **20:4**, 336-357.

Sowa, John F. (1984) *Conceptual Structures: Information Processing in Mind and Machine*, Addison-Wesley, Reading, MA.

Sowa, John F. (2000) *Knowledge Representation: Logical, Philosophical, and Computational Foundations*, Brooks/Cole Publishing Co., Pacific Grove, CA.

Sowa, John F. (2003) Laws, facts, and contexts: Foundations for multimodal reasoning, in *Knowledge Contributors*, edited by V. F. Hendricks, K. F. Jørgensen, and S. A. Pedersen, Kluwer, Dordrecht, pp. 145-184.

Sowa, John F. (2006) Worlds, models, and descriptions, *Studia Logica*, Special Issue *Ways of Worlds II*, **84:2**, 323-360.

Sowa, John F. (2010) The role of logic and ontology in language and reasoning, Chapter 11 of *Theory and Applications of Ontology: Philosophical Perspectives*, edited by R. Poli & J. Seibt, Berlin: Springer, pp. 231-263.