

Assurance Through Reasoning

Deductive Reasoning

MS 670, 1911

Charles S. Peirce*

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[p. 1, 7 June 1911, 5:00pm]

The word “Deduction” will here be used in a generalized sense, to include any necessary, or mathematical, Reasoning—any reasoning of which the premisses,—or, as they will here often be called, the “Copulate Premiss”,—having been asserted, the conclusion cannot consistently be denied by the same assertor. But though all deduction is thus necessary reasoning it will, nevertheless, be convenient to divide deduction into “Necessary Deduction” and “Probable Deduction,” the latter expression denoting any deduction concluding that, understated conditions, a given kind of “Event” would have ~~any p~~ a stated probability, whether the statement of it be numerically precise or be as vague as it may; while by a Necessary Deduction is to be understood a Deduction which simply ~~concludes that it~~ reaches the conclusion

*[Unpublished manuscript MS 670, 1911. This is the later draft of the version under the same title (MS 669) written between 7 June and 17 June 1911. MS 669 was written between 25 May and 2 June of the same year. According to the Peirce Edition Project, “In April 1911 [John Willis] Slaughter set Peirce a reminder [to contribute a chapter to a collection of essays on “Significs” to honor Victoria Lady Welby]. Peirce was at the time trying to write a book on “A Logical Criticism of Essential Articles of Religious Faith” (MSS 846–856), whose first part was to be on “logical critic”. Slaughter’s letter gave him a new impetus, and in early August he announced to Lady Welby that, although he wanted at first to write an abstract of his entire system of logic, constraints of space and time were forcing him to limit himself to “Logical Critics, that is, to the quality of grade of assurance that the three classes of reasoning afford”. MSS 669–670, titled “Assurance Through Reasoning”, may be part of an earlier effort” (extended head note to *Essential Peirce* Vol. 2 at <http://www.iupui.edu/~peirce/ep/ep2/headers/notes/note30x.htm>, accessed 14 September 2004). Original manuscripts are available in microfilm, *Manuscripts in the Houghton Library of Harvard University*, as identified by Richard Robin, *Annotated Catalogue of the Papers of Charles S. Peirce* (Amherst: University of Massachusetts Press, 1967), and in *The Peirce Papers: A supplementary catalogue*, *Transactions of the C. S. Peirce Society* 7 (1971), 37–57.]

that a certain state of things would necessarily result from the facts asserted in its copulate premiss [p. 2, 7 June 1911, 6:05pm] without resorting to any calculation of probabilities. Thus, the ~~reason~~ kind of reasoning ~~here~~ [herein] termed Necessary Deduction is no more necessary reasoning than is Probable Deduction, the reasoning of which, being mathematical, is of course necessary. The ~~former~~ designation here applied to the former ~~is~~ ought, strictly speaking to be, “deduction which concludes a necessary, or apodictic, conclusion, without the introduction of the concept of probability in the process of deriving that conclusion”; but it is presumed that the indulgent reader will be willing to put up with the ~~brief~~ name “Necessary Deduction,” in view of its brevity and of the fact that nothing whatever would be gained in discussing this kind of deduction, by reminding the reader in almost every sentence that there is a complication of Deduction with which, for the time being, he has nothing whatever to do.

~~In considering~~ [The [readers] study of] the reason of the validity of deduction will be great[ly] [p. 3, 7 June 6:40pm] facilitated by practicing the expression of the copulate premises and conclusion of ~~every~~ a variety of deductions ~~that comes under his consideration in such study~~, not in the ~~ordinary~~ syntax of any spoken tongue, but in a certain *diagrammatic syntax* ~~that~~ which has been specially devised for ~~the~~ such purpose, and which shall forthwith be here described.¹ This system supposes that every single word ~~expresses~~ it uses is an assertion. Thus, if the word “man” is put upon ~~the~~ a sheet appropriated to expressions in this syntax, it will be understood to mean “There is a man,” i.e. “Something is a man.” So if “loves” appear on the sheet, it will be interpreted as asserting that [[“]]Somebody at some time did love or will love somebody or something.[[”]] Of course, one cannot *speak* in this syntax, since it is diagrammatic; for nobody can talk a diagram. It is essentially superficial: ~~what there~~ a surface ~~[must be a surface]~~ ~~on which assertions are to be made~~ must be devoted to the purpose of receiving the diagrammatic expressions, and this surface is called The Phemic Sheet. Are ~~they to be~~ there expressions written, drawn, or incised? All three of these methods are employed; though the last is only ~~used in pretence~~ a make-believe incision repre[se]nted by a fine line. Because ~~in expressing~~ we generally both write, and draw, and [p. 4, 7 June 1911, 7:20pm] else [pretendedly] incise, in almost every ~~expression~~ assertion made in this syntax, we cannot say we write it, or that we draw it, or that we incise it. We

¹[p. 3A, 8 June 1911, 1:15pm] I must acknowledge that my assertion that the readers study of the ~~reason of~~ critic of necessary deduction will be greatly facilitated by the ~~study~~ practice of this diagrammatic syntax is merely an analogical inference from my own experience. I had studied and practiced the whole theory of reasoning for many years and perhaps was unconsciously puffed up with my acquisitions as ~~a true~~ the ideal student never will be. All I can say for certain is that after some years' acquaintance with this syntax I ~~was a~~ go found it had taught me a good deal more than I had at all suspected that it could. Beyond that I cannot know how much ~~[it will] teach~~ a reader will gain from similar practice.

say that we *Scribe* it; and that assertoric form to which, when we say we “scribe” it, we mean that we “give it a local habitation and a name,” that we *embody* it, i.e. give it existence in a single “instance,” though in itself it is only a *kind*, and so a “May-be”, not a definite individual person or thing. Just so, the word “the,” though it be printed twenty times on an ordinary page, is only one single word, those twenty occurrences of it being so many *instances* of the single word. In this diagrammatic syntax in which every word is an assertion, the form which *would*, were it scribed ~~would~~ be embodied in an instance, is called a “Graph.” It is a graph whether it be composed of many graphs, or whether it ~~is an atomic~~ be a “Graph-atom.” ~~The sheet upon which it is pretended the student of logic exercises himself by scribing the premises that making believe to scribe he makes believe he asserts is called the “Phemie Sheet.”~~

[p. 5, 8 June 1911, 3:00pm] The fact that according to this syntax any single word is an assertion classes the syntax, but does not characterize it. With that understanding, “man” by itself would most naturally mean what this syntax makes it mean, and the same may be said of “loves,” “gives,” “sells,” etc. Yet “man” might mean “Everything is a man”; and “love,” “everything loves everything” etc. This latter was ~~in fact~~, indeed, the first proposal which led up to the syntax here to be described. ~~The former~~ That first proposed may be named “the Syntax of Entitative Graphs” and that which is here to be described is “the Syntax of Existential Graphs.”

There are but ~~two sign~~ three peculiar signs that the Syntax of Existential Graphs absolutely requires. ~~The first of these is a Sign which shall deny Graph or a scribed assertion. Such a sign is required, since “Not man” would only mean, “There is something besides some man,” or else, “There is something that is not a man.”~~ [p. 6, 8 June 1911, 4:40pm] The first of these is a sign of identity. An ordinary word or abbreviation would not answer this purpose because it would not show what two objects were said to be identical, which is the sole end of a sign of identity. To write one before and the other after the graph of identity could only be suggested by a person who had no experience of the cases in which such a sign is needed. How, for example, should we express “Somebody loves himself”? A practicable suggestion and one which may occasionally be used is to attach duplicate marks of reference, such as asterisks, obelisks, double daggers, sections, parallels, and paragraphs. But a much more diagrammatic method is commonly used. ~~Namely, when every graph except a “Medad,” i.e. one expressing an entire assertion ought to carry a heavy dot, at a point of its periphery to be agreed upon to be settled by a convention to denote each of its “Subjects,” a word used in this syntax to denote indifferently a Subject nominative or any variety of object. If a graph is a “Monad,” that is, has but~~ [p. 7, 9 June 1911, 2:35pm] Namely, every “Simple Graph,” or “Spot,” that is to say, every graph that is not composed of other graphs, ~~ought~~ unless, indeed, it be a “Medad” (i.e. unless it be, in itself, a complete assertion, such

as “rains,” ~~to mean~~ meaning “it rains,” and as such has no subject), ought to carry at a point of its periphery to be decided upon by convention, a heavy dot to denote quite indefinitely, each of its subjects. Thus, “•dies” will naturally mean “Something dies,” this something being denoted by the dot; and •kills• will naturally mean “Something kills something,” the left-hand dot ~~being~~ denoting the killer, and the right-hand one the killed. So •give•s• may very well be used to mean “Somebody gives something so somebody,” the left-hand dot denoting the giver, the right-hand dot the gift, and the dot below the recipient of the gift. For to give is not a simple physical act, like to illumine, to hear, to push, to pull. If it were it would be “dy[a]dic,” that is, the assertion of it would be sufficiently completed by two [p. 8, 9 June 1911, 3:10pm] subjects. But it is a transfer of something created by the mind, to wit, a legal right; and such actions are mostly “triadic,” at their simplest. These heavy dots attached to spots and indefinitely denoting their different subjects are called the “pegs” of the spots. They have their analogues in chemical graphs. Now we have only to stretch such a heavy dot into a heavy line, and it automatically becomes an assertion of the identity of the two graph-subjects denoted by its two extremities. Thus, Fig. 1 asserts ~~that~~ “Mary is a woman.” Fig. 2 asser[t]s “Cain kills Abel.” Fig. 3 asserts “Somebody kills himself.”



Figure 1.

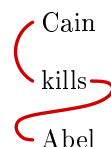


Figure 2.



Figure 3.

This way of asserting identity, along with other important advantages, has that of automatically furnishing *the second general sign required by the syntax of existential graphs*. This is the “Spot of Teridentity.” At first blush it is likely to seem to the reader that the identity of three [p. 9, 9 June 1911, 6:45pm] subjects is nothing but the simple identities of the first with the second, and of this with the third; whence follows the identity of the

first and the third. But in the Critic of Deduction it is needful to think more accurately than that. It is true that if the three can be definitely and distinctly identified, their teridentity is given ~~by~~ in two simple identities. But from the premisses “Some wise man is identical with some virtuous man” and “Some virtuous man is identical with some able man,” it by no means follows that “Some wise man ~~some virtuous man, and~~ is identical with some able man” ~~are coidential~~, much less does it follow that because “Some wise is identical with some virtuous, some virtuous is identical with some able, and some able is identical with some wise”, that some wise, some virtuous, and some able are coidential; and consequently the existence of coidentity is far from consisting in the existence of two or in three simple identities.

It thus becomes evident that we must have a Spot of Teridentity, and that no Existential Graph of Teridentity can be built up of instances of the graph of [p. 10, 9 June 1910 [sic, 1911], 7:30pm] simple identity. However, ~~the~~ a simple point of branching on a line of identity automatically signifies teridentity, as in Fig. 4, and teridentity once ~~scribed~~ conquered we have the means, by it alone, of expressing any grade of multi-identity, as in Fig. 5.

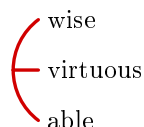


Figure 4.

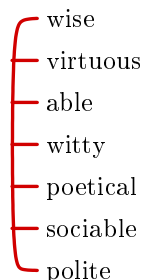


Figure 5.

We thus perceive that the essence of Plurality is not perfectly realized in the number two, while it in three; which, by the way, indicates the truth of those numerous languages of every type ~~which~~ that distinguish the grammatical dual number from plural. Moreover, it furnishes a hint toward explaining why perfectly comprehended and essential logical divisions are ~~never~~ often trichotomies, ~~and are only give~~ but never result in a number of parts ~~by two dichotomies~~ having any prime factor greater than three. There is more to be said about the Line of Identity, after the third ~~essential~~ indis-

pens[a]ble peculiar sign of the Syntax of Existential Graphs has been [p. 10 11, 10 June 1911, 1:15pm CSP: “This an the two next sheet were wrongly numbered”] considered.

The third is one that shall deny a Graph-instance, or scribed assertion. For without that we could not, in Existential Graphs, express any proposition that form which is, *par excellence*, Critical; namely, the conditional form, “If Antecedent then Consequent.” No more could we in Entitative Graphs ~~where we set out with the condit~~ (whose development starts from the conditional form) express a proposition of the particular form with which the development of Existential Graphs begins. But that Conditional Proposition just written may be expresses by the aid of a sign of Negation in the form, “That the Antecedent is true and the Consequent not true, is not true.” This example shows us that along with the sign of negation we require one of those *Collectional Signs*,—“Klämen,” Schröder calls them,—such as in algebra are the parentheses, brackets, braces, the vinculum, ~~and, in the principal effect~~ [period -green]. ~~the only indispensable special signs of~~ [p. 11 12, 10 June 1911, 1:55pm] The functional signs when more than single letters are attached to them belong to this class of Collectional Signs, which class is the most important,—it would, indeed, be strictly true to say they are the only indispens[a]ble,—signs of algebra. But the whole of the strict truth is, in this case, not important. What *is* important is to understand that the essential power of algebra is due exclusively to collectional signs. It will not be surprising, then, to find that logical critic cannot be firm upon its legs and able to progress until it is provided with convenient collectional signs. However, they are only first needed in Existential Graphs ~~when~~ after this syntax has been provided with a sign of negation; for whether one says “A is true, and B is true, and C is true,” or says, “A and B and C is true,” or says “A and B are true and C is true,” ~~is the same thing or says “To assert “A” is to speak truly, and to assert “B” is to speak truly and to assert “C” is to speak~~ [p. 12 13, 10 June 1911, 3:20pm] “To assert that the assertion of A would be true, and that the assertion of B would be true, and that the assertion of C would be true is to assert nothing but the truth,” or whether one says “To assert that the assertion of C would be true is to assert nothing but the truth,” or whether one says “To assert that the assertion of A would be true and that the assertion of B and C together would be true,” or whether one says, “To assert that the assertion of A and B together would be true and that the assertion of B and C would be true” is perfectly indifferent: they all come to the same thing.

On the other hand to say that “To assert that the assertion of A would be *false*, and that the assertion of B would be false, and that the assertion of C would be false” would be to assert nothing but the truth,” would amount to much more than either to say “To assert that the assertion of A would be false, and that the assertion of B and C together would be false, would be to assert nothing but the truth” or to say “To assert that the assertion

of A and B together would be false and that the assertion of C would be false would be to assert nothing but the truth” would be very different. But while the syntax of existential graphs thus needs [p. 13, 14, 10 June 1911, 10:00pm] both a sign of negation and an endless series of collectional signs, there is no reason why a single sign should not perfectly fulfill both these purposes. Here immediately follows an explanation of this sign as it is seen by the minds eye; and this shall be supplemented by an account of how this ideal sign can be conveniently represented on paper.

Let the descriptions of things or of events of each of which one wishes to say that something of the sort exists or sometimes occurs be scribed in black upon the white surface of the phemic sheet. Then if of some description of object one wishes to deny that it ever exists, let a part of the sheet be blackened but leaving with white spot left in it, so shaped as to compose in their entirety an instance of a description in question, and let the interpretation be that no such thing or event is ever actualized as that description in its entirety. Thus, Fig. 6 will affirm A and affirm B while denying C and denying D. But Fig. 7, while affirming A and B will only deny that *both* C and D are true; that is it will assert that ~~either~~ no C ever occurs ~~or else that~~ at the same time, or occasion, as D. On precisely the same principle Fig. 8 must be interpreted as asserting that if A is ever true either C or D will sometimes be true; for this is precisely the same as to assert that to say that A is sometimes true and that C and D are each sometimes false would be to assert something not altogether true.

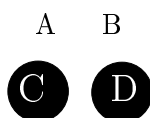


Figure 6.

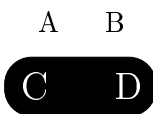


Figure 7.

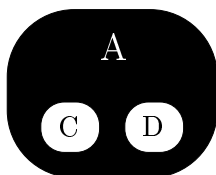


Figure 8.

It is needless to say that the process of making a black surface with a diagram in white upon it is insufferably inconvenient. For that reason in practice one substitutes Fig. 9 in place of Fig. 8.²

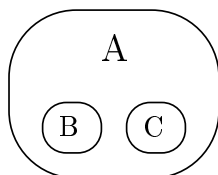


Figure 9.

This sufficiently overcomes the inconvenience of making the diagrams; [p. 16, 12 June 1911, 5:40pm] but it is accompanied by a serious danger. For though the bounding ovals be drawn in as fine lines as they may,—and they must be so drawn,—one is but too apt to think of these inclines as signs, like the graphs, and to forget that they have no meaning at all, but are mere boundaries between black and white. They are called “cuts,”—that is, girdling-edges; for the black areas usually carry white areas within them, and may be thought of as taking away the presence of that which the graphs they carry denote. It is only the color of the area itself which ~~asserts or denies anything as~~ has the force of affirming, if it be white or *evenly-enclosed* (i.e. is enclosed by an even number of cuts, or by none) or of denying if it be shaded or *oddly-enclosed*.

It is a help to shade the oddly-enclosed areas and omit the lines that represent the cuts, as in Fig. 11, which is equivalent to Fig. 10.

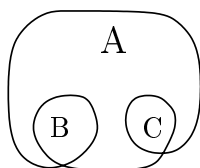


Figure 10.

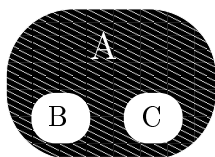


Figure 11.

²[In Fig. 9 Peirce uses A, B and C as spots, whereas in Fig. 8 they are A, D and C. The difference is immaterial.]

Though the cuts in themselves have no meaning whatever, but are, [p. 17, 12 June 1911, 6:15pm] at most, mere punctuation-marks separating “particular” affirmations from general negations, yet ~~the~~ an *enclosure*, that is, ~~the~~ an area together with all that it carries, including whatever other ~~area~~ ~~or areas~~ enclosure or enclosures it may carry is a graph-instance. The area upon which a given area lies is called *the place* of the latter area, or *the place* of the cut which is its outer boundary; while an area within a cut ~~may be~~ is sometimes called *the area* of the cut. The “place” of any graph-instance is the outermost of the areas which any part of it enters. But an instance of a simple graph, i.e. one not having parts that are themselves graphs, must be wholly in one area: and this is not an unimportant fact. Let us now exercise ourselves in interpreting a few graph-instances. Fig. 12 reads “If it thunders it must have lightened”; for it denies an occurrence of thunder without [p. 18, 12 June 1911, 7:10pm] its having lightened.

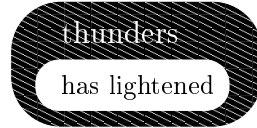


Figure 12.

Fig. 13 reads “If there is anything there is a mind.” Fig. 14: “If anything is possible there must be a mind.” Fig. 15: is an absurdity; for it denies that which does not assert anything. Fig. 16: “Either there is a mind or there is an absurdity.”

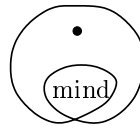


Figure 13.



Figure 14.



Figure 15.



Figure 16.

This may be considered as asserting that the existence of a mind is an absolute necessity. It does not quite say this because it does not say there *would* be a mind under all circumstances. ~~For that purpose~~ To express that explicitly, it would be necessary to indicate, upon the very outermost, or border, area, the logical universe is that of metaphysical possibility, so that oddly-enclosed graphs express necessities. The nature of the universe or universes of discourse (for ~~there~~ several may be ~~several~~ ~~[may be]~~ referred to in a single assertion) in the rather unusual cases in which such precision is required, is denoted either by using modifications of [p. 19, 13 June 1911, 10:25am] the heraldic tinctures, marked in something like the usual manner in pale ink upon the surface, or by scribing the graphs in colored inks. In the former method it is usual to employ the different metals ~~to mark~~ (or, argent, fur, and plomb) to ~~signify~~ mark the different kinds of existence or actuality, the different colours (azure, gules ~~rouge~~, vert, purpur) for the different kinds of possibility,—possibility consisting of ignorance, of variety, of power, of futurity; and the furs (sable, ermine, vair, potent), for the different kinds of intention.

But when what is scribed has not to go to press, nothing else is so simple as the use of colored inks, of which excellent ~~violet~~ mauve, crimson, scarlet, maroon or reddish brown, buff or yellowish brown (the so-called bismarck brown) olive or greenish brown (in *appearance*; for it is really nothing but darkened gamboge), blue, violet, and royal purple, are ~~easily~~ easy to be had. The pale colors may be used for possibility, the dark for actuality, and the high for [p. 20, 13 June 1911, 11:00am] intentions. To express the metaphysical necessity of mind, one may on a pale reddish ground make a deep red area bearing ~~the word man a pale~~ a white spot in its turn on which is written “mind.” This will express, “It is metaphysically impossible that there should not actually exist a mind.” But it is to be remembered that an assertion, — although in itself, either a command, usually obeyed (like other ~~customary~~ commands that the interpreter is ~~habituated~~ accustomed to obey), by force of habit; or else, taken as the totality composed of the appearance of the assertor, and all the interpreter’s experience of him, of that assertor’s tone and manner, together with the proposition considered

as asserted as and by whom it is, ~~it is~~ to be regarded as a retroductive argumentation tending to persuade its interpreter of the ~~breadth~~ reality of its substance,³—yet, when it is to be made the premises of a yet undeveloped necessary deduction, it ought to be regarded merely in its syntactical *form*, carrying significant matter, it is true,—but that matter packed up as concisely as it may, and not to be undone and scrutinized until the deductive conclusion shall have been drawn, so that the energy of attention may not be wasted upon it.

Every part of the ~~composite~~ copulate premises that does not need to be dissected for the purpose of drawing the conclusion is best represented by a single letter with the requisite pegs about its periphery; while at the nodal point of the argumentation no subtilty can be useful in drawing any part of the conclusion ought to be spared. But before we consider ~~the~~ deductions, we need to exercise ourselves in ~~express~~ correctly so scribing graphs that in- [p. 22, 13 June 1911, 12:23am] volve both enclosures and lines of identity as to express various familiar forms of thought.

In the first place, if the reader can put up with another technical term, it must be remarked that the line of identity, that heavy kind of line that when evenly enclosed affirms that that individual object that is denoted by one of its extremities *is identically the same as* the individual denoted by its other extremity. Since no concept stands less in need of analysis in the process of deduction that does that of identity, it ought to be regarded as a *simple graph* expressing that “Something (denoted by one of its extremities) *is the same individually as*” (that which its other extremity denotes[]); and consequently it must be scribed, if at all, upon some single area within the border of the phemic sheet. Any instance of it is, however, scribed as a continuous line, which is divisible into and [p. 23, 13 June 1910 [sic, 1911], 6:00pm] consists of as many parts as one may like. Now it is not the question how it may be with a real line; for let us assume that our conception of a line is correct, or, at any rate, is all we are talking about. Then there can be no question that there is room upon it for a multitude of points at least as great as the multitude of all numbers, rational or irrational, between any two values,—say between 2 and 4.

The mathematicians assume that any simple line *is composed* of just so many points, and no more. The present writer ventures to disagree with them;—and let it be well understood that, as just said, it is a disagreement in regard to our notion of a line. His reason is that if we cut a line in two thus making two new ends, our notion is that there will be a point at each of those ends while if we cut the series of all real numbers in two so as to leave a number on one side next to the cut, then on the other side however close [p. 24, 14 June 1911, 1:10pm] to the cut any number may be there will

³By the “Substance” of an assertion is to be ~~meant~~ [p. 21, 13 June 1911, 11:45am] understood that state of things or event whose Reality it asserts.

be an *endless* series of numbers yet nearer. For example if we cut the series of numbers so that 3 shall be the highest of all the numbers below the cut, then there will be above it 3.5 3.05 3.005 3.0005 and so on *endlessly*. For 3.00000 *et cetera ad infinitum* 1, is the same as 3, or, if it be not, then the number expressed by substituting 0001 for the last 1 is higher than 3 and lower than 3.00000 *et cetera ad inf.* 1. No competent mathematician will hesitate to endorse the statement that if the series of real numbers be cut so that there shall be a number nearest to the cut on one side of it, then the series as one approaches the cut on the other side is *endless*, whether the series embraces all real numbers or only all real rational numbers. It is one of the recognized truths of arithmetic, demonstrated by Euclid.

The cause of the misrepresentation by mathematicians of the concept of a line is ~~simply~~ [p. 25, 14 June 1911, 1:30pm] that their representation is faithful to every measure of line; or at least is so nearly as faithful to the concept of a measured line as to result in no measurable error, and they would not correct their concept without going into logic deeper than it is pertinent to measurement to go. But Kant sometimes defines the continuity of time in a manner which solves the difficulty and is perfectly applicable to a line, although Kant himself fails distinctly to appreciate the merits of his definition. He says that the continuity of time consists in the fact that every part of a time is a time; that is time is not wholly nor partially composed of instants.⁴ The continuous parts of a line, i.e. where there is neither a node, or point of branching, nor an extremity, nor any marked point, contains no point. But as soon as a point is marked upon a line the continuity is broken at that point and no- [p. 26, 14 June 1911, 2:05pm] where else; and a point on a line occupies no part of the line. To be sure, if you choose to *call* a point a part of a line, by an arbitrary misuse of the word "part," then you must define the continuity of a line differently. You may say that every *constituent part* of a continuous line is a line. Or you may say that every *constituent part* of a continuous line is a line. Or you may say that every part of the *room* on a line affords ~~has~~ *room* for a line. This is merely a variation of nomenclature. The multitude of rational numbers is demonstrably no greater than the multitude of whole numbers; but the multitude of real irrational numbers is greater.

There are still greater and greater multitudes endlessly. The reason the mathematician can conceive of no more points on a line is not at all that there is no ~~more~~ room for more on the line; for ~~any~~ no multitude whatever of points can fill any linear space; but the reason is that he has exhausted his vocabulary for distinguishing the units of a multitude. He can only approximate to distinguishing the irrational numbers, and for distinguishing the different individuals in higher multitudes ~~he is~~ [p. 27, 14 June 1911, 3:57pm] than that of the irrational reals, the human mind is not capable

⁴C.r.r.V. 1st Aufl. S. 169ff. [Immanuel Kant, *Critique of Pure Reason*.]

even of approximating to doing so.

The line of identity does no more than assert the individual identity of the objects denoted by its extremities. Identity is a dyadic relation; and among dyadic relations it belongs to the type to which belongs the relative term “is loved by whatever loves.” ~~One of the~~ For to say that “some woman is identical with something that is beautiful” is precisely the same as to say that “some woman is identical with ~~something~~ whatever there may be that a certain something that is beautiful is identical with.”

The term “line of identity” is taken in such a sense as to limit it to *graphs*. Now a graph-instance must be scribed in some one area and be limited to that. But a line of identity may be continuous with another line of identity in another area, as in Fig. 17; and [p. 28, 14 June 1911, 5:00pm] such a series of lines of identity thus continuous each with the next is called a *ligature*. ~~This may be read “Something is a w” “Some woman is identical”~~



Figure 17.

Fig. 17 may be read, “Some woman is identical with something which is not identical with anything unless it be with something that is other than everything except what is other than anything beautiful,” which comes to saying, “Some woman is other than everything beautiful,” or “Some woman is not beautiful.” ~~Thus it will be seen that the line of~~ Thus a ligature which has its outer end evenly enclosed and its inner end oddly enclosed has no other effect on the whole than asserting the identity of something denoted by a graph in the outer area with some thing that is *not* of the description signified by a graph in the oddly enclosed area.

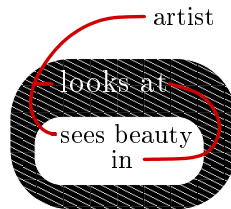


Figure 18.

Fig. 18: “Some artist sees beauty in whatever he looks at.” Fig. 19: “Some man sees beauty only in things he looks at.” ~~All the signs that essential to the Syntax, or the connexion between the indivisible gr have now been illustrated. Let us now consider two or three~~

[p. 29, 16 June 1911, 5:45pm] All the strictly ~~essential~~ synthetical ele-

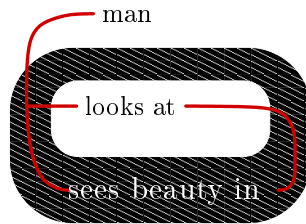


Figure 19.

mentary signs essential to this syntax have now been explained. But the writer has been led by fourteen or fifteen years experience of it to rate its utility so very high that he will here insert examples of all the propositional forms up to a certain limit of complexity, so as to familiarize the reader with the use of it, and ~~will then~~ following these will make the briefest possible restatement of the definitions of the signs.

It will be understood that the system is not a complete syntax, but is only a *syntax of assertions*. It is a question whether an assertion should be regarded as ~~belonging to~~ a command to believe, that is, as an action on the part of its Utterer,—a word which will throughout this writing be applied as much to one who puts forth a sign by writing or by any other kind of action as to one who does so [p. 30, 16 June 1911, 6:20pm] by means of his voice,—as an action, then, on the part of its Utterer with the deliberate purpose of causing belief in the mind of its Interpreter, through a habit in that mind, much in the same way as an officer's commands causes a soldier to obey with very little reflexion, if any at all; or whether, on the other hand, the assertion ought not rather to be ~~considered~~ regarded as the voluntary manifestations of the utterers belief usually, but not necessarily, with more or less hope that the fact of belief so manifested will appeal to the interpreter's reason as a sufficient premises to make him conclude that it is true; or whether finally it should be considered, neither as a command nor the manifestation of a premises of reasoning, but rather as a third ~~genus~~ species of sign more or less resembling each of those others.

In order to solve this far from easy problem, let us turn our attention [p. 31, 17 June 1911, 1:20pm] back to the essential nature of a *Sign*, in general, and consider whether that does not point out to us the nature of that fundamental division of signs which will ~~yield~~ furnish us with the concept of the Assertion as one of its members. A Sign, then, is anything whatsoever,—whether an Actual or a May-be or a Would-be,—which affects a mind, its Interpreter,—and draws that interpreter's attention to some Object (whether Actual, May-be, or Would-be) which *has already* come within the sphere of his experience, and beside this purely selective action of a sign, it has a power of exciting the mind (whether directly, by the image or the sound or indirectly) to some kind of feeling, or to effort of some kind

or to thought; and so far as it ~~does~~ has any such effect *quâ* sign—for besides being a sign, it may also be a music,—but so far as it excites feeling, will, or thought *as a* sign, it [p. 32, 17 June 1911, 2:45pm] connects the feeling, will, or thought in the mind of the Interpreter with its Object as due to it, as the interpretation of it.

The writer is not altogether satisfied with this attempt to analyze the nature of a sign; but he believes that the sign calls up its Object or Objects, for there may be several, and besides that excites the mind as if it were the Object that had this effect. If a person reads ~~a statement~~ an item of news in a newspaper, its first effect on his mind will probably ~~depend on his habits; upon someone upon anyone, perhaps, of little experiences~~ its first effect will be to cause him to believe the truth of what is asserted, that is, to believe the reality of the substance of it; upon another the first effect something that may conveniently be called an “image” of the event, without any judgment as to its reality.

ASSORTED PAGES

[A thr R, p. 7, 8 June 1911, 5:05pm] a single subject, such as “man”, “dies”, etc. it makes no difference ~~where~~ [at what point of its periphery] the dot is placed. If it be a “Dyad”, or Graph of two subjects, the Subject Nominative, or more ~~existential, and~~ [actual or] active of the two should usually be put to the left, and the more ~~potential~~ passive to the right, or the more concrete may be placed in differently to the right or left, and the more abstract, indifferently above or below; as, •kills•, and $\bullet \overset{\cdot}{I} = \bullet \overset{I}{\bullet} = \overset{\cdot}{I} \bullet = \overset{I}{\bullet} \bullet$. If there be a Subject nominative, direct object, and indirect object, the dots ~~represe~~ denoting these may be placed respectively to the left, to the right, and below (or above); and so on.

[[On verso, in red ink, and in Peirce’s hand:] “Please do not fail to note the ‘N.B.’ on the other side of this leaf, before opening the package I send”. [Below, in blue pen,] “J is going to pick sxxraubrys”. [Also twelve sketches of graphs in light pen]

[A thr R, p. 12, 10 June 1911, 3:10pm] “To assert that A is true, and that B is true, and that C is true is to assert only the truth”, or whether one says, “To assert that A and B [together is] ~~are~~ true, and that C is true is to assert only the truth”, or whether one says “To assert that A is true and that B and C [taken together is] ~~are~~ true is to assert only the truth” is perfectly indifferent: they come to the same thing. But “To assert that A and B taken together is not true, and I

[A thr R, p. 23, 13 June 1911, 12:45pm] consists of as many parts as one may like; so that [it might be urged that] what it actually expresses something

